The Impact of Risk-Based Pricing and Refinancing on the Student Loan Market

Natalie Bachas *

Abstract

Advances in data-driven underwriting have both efficiency and equity implications. In the $1.3 trillion student loan market, private lenders offer a growing distribution of risk-based interest rates, while the federal loan program sets a uniform price. I measure changes in consumer surplus that occur as low-risk types refinance out of the government pool into the private market. I use a dataset from an online refinancer to estimate a structural model that relates borrowers’ repayment choices to interest rates. I estimate refinancing increases low-risk surplus by $1,302, and show substantial distortionary costs (32% of the average transfer) under a pooled, uniform interest rate. To maintain access to the current uniform rate, the government must subsidize high-risk borrowers $1,507 on average.

* Affiliation: Stanford Institute for Economic Policy Research (SIEPR) & Department of Economics, Princeton University. Email: nmcox@stanford.edu. I thank my committee members, Ben Handel, David Sraer, Ulrike Malmendier and Emmanuel Saez, who provided invaluable advice throughout the research process. This work has benefited particularly from the comments of Alan Auerbach, Pierre Bachas, Zarek Brot-Goldberg, Claire Celerier, Kei Kawai, Patrick Kline, Jon Kolstad, Jennifer Kwok, Julien Lafortune, Waldo Ojeda, Jon Schellenberg, Avner Schlain, Katalin Springel, Boris Vallee, Danny Yagan, Gabriel Zucman and seminar participants. I also thank the patient and helpful employees at the firm whose data I analyze.
1 Introduction:

Risk-based pricing has increased in use and sophistication, especially as more lenders and insurers transition to an online setting. It has been shown in theory and practice that the ability to identify and accurately price consumer risk can generate large efficiency gains in markets with selection.\(^1\) However, if advances in risk-based pricing create clear winners and losers, this will also change the government’s role in ensuring equity and redistribution of surplus.

This is especially true in the student loan refinancing market, where private online firms use rich financial and educational data to underwrite student borrowers who have finished schooling. These individualized prices contrast with those of the Federal Direct Loan program, which offers borrowers a uniform interest rate despite observable variation in the expected costs of lending. Moving from average to marginal cost pricing could correct allocative inefficiencies - low risk types might increase borrowing when faced with an undistorted price in the private sector. But pricing innovations could also have complex implications for how private and public lending options coexist. As low risk types refinance into the private sector, the average risk of the remaining federal borrowers will rise, forcing the government to either raise its uniform rate or subsidize the remaining pool.

This paper studies this efficiency-equity tradeoff empirically, using a dataset of applicants from an online student loan refinancing firm that employs comprehensive risk-based pricing. Using a series of firm-conducted price changes, I show that observationally similar borrowers are interest rate sensitive during repayment: they increase monthly payments and shorten maturity when interest rates increase. This suggests that there will be a distortionary “cost” to the governments’ one-size-fits-all pricing policy as it transfers from low to high risk borrowers. I estimate and combine this monthly payment elasticity with the observed distribution of risk based prices to quantify the deadweight loss and redistribution that occurs under a uniform interest rate, as well as the budgetary impact of

\(^1\)Several papers (Einav, Jenkins, Levin, 2013; Einav, Finkelstein, Levin, 2010; Edelberg 2006; Paravisini, Schoar 2013, and Phillipon 2016) show that credit-scoring can generate both efficiencies in consumer lending, and impact market structure. There is also a literature that studies uniform and average cost pricing schemes in the presence of heterogeneous risk (Bundorf, Levin, & Mahoney 2012; Einav, Finkelstein, & Cullen 2008; Hurst, Keys, Seru, & Vavra 2015) in several markets (health insurance, mortgages) that has shown that while uniform pricing policies achieve cross-sectional redistribution, they can also distort consumer choices and generate welfare loss.
low risk borrowers refinancing into the private sector.

The dataset I use provides precise, borrower-level information on risk-scores, interest rates, repayment decisions, demographics and household balance sheets. It also contains 10 interest rate changes that were conducted at a firm-wide level to gather quasi-experimental evidence on maturity and application elasticities. I leverage this variation in price schedules to measure how interest rates impact the loan maturity decisions of observationally similar borrowers.\footnote{Loan maturity is perhaps the most fundamental decision made by the borrower during repayment, given that the choice of how much to borrow is already fixed. It is the only means a borrower has during repayment to lower monthly payments and increase immediate liquidity.} Borrowers reveal their intertemporal preferences when making a maturity decision: by extending maturity they reduce their monthly payment, but increase their total interest paid.

Reduced form evidence reveals that the borrowers in my sample decrease maturity when interest rates increase, and that this sensitivity increases with borrower quality. This suggests that under a uniform average cost interest rate, low risk borrowers will make inefficiently high monthly payments, and high risk borrowers will make inefficiently low monthly payments. I next write down a model where borrowers choose a maturity to maximize expected utility; this model maps the reduced form maturity elasticity to the underlying parameter of interest, the intertemporal elasticity of substitution (IES), and allows for a rich counterfactual analysis. I use the first order condition that captures the maturity tradeoff to estimate the model using non-linear least squares and find a moderately high average IES of .6.

Using the estimated repayment model, I compare the size and distribution of borrower surplus under several pricing regimes: full pooling under a break-even uniform rate, pooling with a refinancing option using FICO-based pricing, and pooling with a comprehensive risk-based refinancing option. The first average-cost interest rate policy redistributes roughly $1,200 from low to high risk borrowers, but generates an average distortionary cost of $450, or 32% of the average transfer. The policy achieves more modest redistribution over income quantiles given that borrower risk type is an imperfect proxy for borrower income. When low-risk types have the option to refinance into the private sector they gain on average $1,500 in surplus – they face a lower absolute level of interest rates, and respond by making smaller monthly payments. Both the distribution of risk based interest rates and the gains to low risk borrowers are much larger when
firms price on borrower characteristics, like savings, income, and education, in addition
to FICO score. This suggests that non-traditional scoring methods can benefit individu-
als who are low risk, but have underdeveloped credit histories (i.e. the student borrower
population).

These findings highlight how developments in the private sector’s ability to price bor-
rower risk will simultaneously i) improve welfare for low risk borrowers and ii) increase
sorting of low risk borrowers out the public repayment pool. I analyze one government
policy response that would prevent unraveling and maintain equity in the federal pool:
providing a net interest rate subsidy. My model highlights how the effective costs of an
interest rate subsidy can deviate from the mechanical costs once refinancing and maturity
responses are accounted for. For example, lowering the uniform interest rate will reduce
refinancing into the private sector, improve the average risk of the remaining federal
borrowers, and therefore decrease the subsidy’s average cost.

This paper relates to several literatures: first, it contributes research on how borrowers
finance their higher education with student loans. While work has primarily focused on
optimal borrowing limits and repayment structures (Lochner and Monge-Naranjo 2015;
Rothstein & Rouse 2011; Avery & Turner 2012; Yannelis 2015; Lucas & Moore 2010,
Beyer, Hastings, Neilson, & Zimmerman 2015), I consider how interest rates can be used
as a policy instrument. The question of how borrowing and consumption decisions re-
spond to interest rates and are mediated by credit constraints is central to the household
finance literature (Adams, Einav & Levin 2013; Gross & Souleles 2002; Martins & Vil-
lanueva 2006; Karlan & Zinman 2005; Summers 1981). Several papers look specifically
at maturity choices (Attanasio, Goldberg, & Kyriazidou 2008; Hertzberg, Liberman, &
Paravisini 2016), and also find that long maturities are preferred by riskier, liquidity
constrained borrowers. Finally, my paper contributes to a literature that structurally
estimates parameters relating to risk aversion and consumption smoothing using micro-
data on consumer choices and quasi-experimental variation in prices (Einav & Cohen
2007; Handel 2013; Best et. al. 2015; Gruber 2008).

The rest of the paper proceeds as follows. Section II describes the setting and data
with an emphasis on the variables impacting borrowers’ maturity choices, and the use of
exogenous interest rate variation to identify maturity elasticities. Section III describes
the theoretical framework, Section V estimates the maturity demand model and discusses
the results. Section IV outlines a welfare framework and analyzes several counterfactuals. Section V concludes.

2 Setting and Data

2.1 Institutional Background

While this paper focuses on the repayment of student loans, it is necessary to understand their origin. Loans that come from the federal government are by far the most popular option to finance post-secondary education – over 90% of the student loan market consist of Federal Direct Loans. This paper concentrates on federally originated loans that may subsequently be refinanced in the private sector.

There are two key loan-related facts that motivate my paper: growth in the originated volume of Federal loans and heterogeneity in borrower risk. Origination rates in the Direct Loan program, where there is no risk-based underwriting and generous lending limits, have skyrocketed over the past decade. The outstanding volume of student debt has quadrupled in the last 12 years, and the median borrower’s holding has grown from $14,000 to $19,500 (Looney, Yannelis 2015), leaving many borrowers with sizable monthly payments and large amounts of accumulated interest.

Growing in line with origination rates have been average delinquency rates – the average three-year cohort default rate (CDR)\(^3\), peaked at 14.7% for the 2010 cohort, compared to a rate of 5.2% in 2002. However, these average trends mask important heterogeneity: default rates amongst graduate students and individuals at 4 year institutions have remained consistently low. These relatively low-risk borrowers make up the majority of the dollars lent by the Direct loan program – graduate students are some of the biggest borrowers, holding 33% of dollars outstanding. This means a large portion of the Direct Loan Portfolio will be “overpriced” by a break-even uniform interest rate.

2.1.1 Student Loan Repayment Options:

Repayment of federal debt, and importantly choice of repayment plan, does not occur until a student has finished schooling (either undergraduate or graduate school).\(^4\)

\(^3\)The percentage of loans in delinquency 3 years after entering repayment

\(^4\)
Students also have the option of changing repayment plans as time progresses. Federal repayment plans fall into two general categories: fixed payment plans, which adjust the monthly payment level to ensure that the full amount of the original loan will be paid off in a specified number of years, and income-based plans, which scale the monthly payment in proportion to the borrower’s income. A CFPB analysis (Gibbs 2017) of the Consumer Credit Panel found 40-50% of borrowers had fully repaid their loans within 5 years, while 25-30% of borrowers took longer than 10 years.

A new repayment option that is growing in popularity consists of refinancing Federal debt in the private sector. Refinancing can take place at the beginning or in the midst of a repayment schedule. Federal loans, which do not carry a pre-payment penalty, are paid off by the private firm which takes over the servicing and liabilities associated with the loan. Student loans that are refinanced in the private market are still not dischargeable in the case of bankruptcy. The majority of refinancing firms are online lenders, who digitally link to applicants’ financial accounts and credit reports, and use extensive amounts of data to assess their risk. Online applications reduce the frictions of refinancing both for the lender, who face lower underwriting costs, and for the borrower, who can apply and review competing interest rate quotes in a matter of minutes.

2.1.2 Interest Rates and Loan Costs:

When choosing a loan maturity, borrowers trade off between the payment made each month towards principal and interest, and the total interest cost paid over the life of that loan. A longer maturity loan will have a lower monthly payment, but a higher overall interest cost (as more interest accumulates, at potentially a higher rate). This means that different loan maturities will appeal to different types of borrowers. For example, individuals with lower incomes who are more liquidity constrained may prefer a long maturity with lower monthly payments.

The federal government charges a single interest rate for all loan terms, whereas the

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4 In most cases a student can postpone repaying their debt for more than a 6-month grace period after graduation without incurring interest.

5 A description of the federal repayment plan options currently available is provided in the Appendix.

6 During my analysis I assume that term choice in the Direct Loan program is “flexible” – this somewhat understates the gains borrowers receive in the private market where there is flexible term choice, and is a generous assumption since over 50% of borrowers in the Direct Loan Portfolio remain in the 10 year fixed maturity plan.
private sector charges a higher rate for longer maturities. Holding maturity constant, the private sector can also offer either a lower or higher interest rate depending on the borrower’s expected risk. Low risk borrowers can therefore decrease both the monthly and total cost of their loan by moving to the private sector and refinancing at a lower interest rate. For higher risk individuals who would actually face a higher interest rate under risk-based pricing, the uniform federal interest rate is preferable and refinancing will likely not occur.

In the private refinancing sector, the variables that determine an individual’s risk-based interest rate are proprietary and company-specific, but are primarily driven by income, spending patterns, degree, income, savings, FICO, and debt. Federal interest rates follow a formula specified under the Higher Education Act. Each year they are determined by an index rate plus an add-on margin that varies by loan type (see Table 1). Graduate students face higher federal interest rates than undergraduates; the fact that they are some of the largest, lowest risk borrowers makes them a population especially prone to refinancing in the private sector.

<table>
<thead>
<tr>
<th>Loan Type</th>
<th>Borrower Type</th>
<th>Index</th>
<th>Add-on</th>
<th>Fixed Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Un/Subsidized Loans</td>
<td>Undergraduate</td>
<td>10 Yr Tr + 2.05%</td>
<td>3.4-4.66%</td>
<td></td>
</tr>
<tr>
<td>Direct Unsubsidized Loans</td>
<td>Graduate/Professional</td>
<td>10 Yr Tr + 3.60%</td>
<td>5.41-6.8%</td>
<td></td>
</tr>
<tr>
<td>Direct PLUS Loans</td>
<td>Parents&amp; Graduate</td>
<td>10 Yr Tr + 4.60%</td>
<td>6.4-7.9%</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: 2011-2015 Interest Rates on Federal Direct Student Loans


Undergraduates are able to borrow at lower interest rates up to a certain limit ($5,500 to $7,500, depending on their year in school), and then must borrow at higher interest rates.

2.2 Dataset:

I use a proprietary dataset from a student loan refinancing firm that contains extensive information on interest rates, risk score inputs and outputs, and maturity and refinancing decisions. The dataset describes individuals who both decide to refinance in the private sector, and on the wider population of those who apply and view an initial interest rate quote. While the first group is very low risk, the second sample allows me to measure the distribution of market-priced risk, and the refinancing propensity, of a more representative sample of Federal borrowers.

Legally prohibited risk-based pricing factors under the Equal Credit Opportunity Act are: race, color, religion, national origin, sex, marital status, age, and receipt of income from any public assistance program.
The main dataset that I use when measuring maturity elasticities is a repeated cross-section of all new borrowers refinancing with the firm over the period of a year; it links background financial information (debt amount, income, assets, credit score) about borrowers with the menu of interest rates they faced and the ultimate maturity choices they made when refinancing with the firm. In the counterfactual section I use a similarly structured dataset which includes all applicants to the firm, including those who do not necessarily choose not to refinance. The sections below describe the data and price variation used during estimation.

2.2.1 Descriptive Statistics

Refinancer Population: The population of borrowers who ultimately refinance are high income, high debt, and highly educated. The median borrower has over $50,000 in student loans, and the median monthly payment on refinanced debt is $600 per month. The median borrower also holds $38,000 in assets, $0 in investments (the 75th percentile has $15,000 in investments), owes $89,000 in additional liabilities, and has a median monthly free cash flow (post tax income minus fixed monthly payments like housing) of $3,100. Borrowers hold a host of degrees and occupations; JDs (lawyers) make up 13% of the sample, MBAs are 17%, MDs (doctors) 5%, pharmacists 6%, and dentists 4%. The majority of borrowers finished school in the last 4 years, with 25% graduating in 2016 and 50% since 2012; however, some are refinancing older loans, with 25% of borrowers having graduated before 2010.

<table>
<thead>
<tr>
<th>Borrower Summary Statistics</th>
<th>Loan Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>Maturity (Months)</td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>75,879, 68,304, 39,799</td>
</tr>
<tr>
<td>Loan Amt</td>
<td>Monthly Payment</td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>67,078, 50,656, 52,890</td>
</tr>
<tr>
<td>FICO</td>
<td>APR</td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>782, 787, 36</td>
</tr>
<tr>
<td>Mortgages</td>
<td>Variable Rate</td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>0.40, 0.00, 0.60</td>
</tr>
<tr>
<td>FCF</td>
<td></td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>3.636, 3.100, 2.574</td>
</tr>
<tr>
<td>Graduate</td>
<td></td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>0.70, 1.00, 0.50</td>
</tr>
<tr>
<td>Age</td>
<td></td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>32.60, 31.00, 6.80</td>
</tr>
<tr>
<td>Dependents</td>
<td></td>
</tr>
<tr>
<td>Mean, Median, SD</td>
<td>0.50, 0.00, 0.90</td>
</tr>
</tbody>
</table>

This table contains summary statistics describing the population of borrowers who ultimately refinance their loans. The upper rows describe the income and characteristics of the borrowers themselves. Income refers to yearly post-tax income. FCF refers to the monthly post tax income minus all fixed expenses. Graduate refers to the portion of the population who has a graduate degree. The lower rows describe the terms of the refinanced loans. Variable rate is the proportion of loans that have a variable interest rate.

The impressive background of these candidates translates into them obtaining consid-
erably lower interest rates when refinancing. The average previous federal interest rate on the loans (before refinancing) was 6.7%, and the average refiner saved 2.21 percentage points when refinancing.

**Applicant Population:** In addition to refinancers, I also observe a larger, less selected sample of website visitors who see an interest rate quote but do not necessarily proceed with the refinancing process. This sample is more representative of the population of graduate student borrowers who have federal loans. In the Appendix (Figure 11), I compare the debt and income quantiles of my applicant sample to a nationally representative sample of graduate student borrowers – they look very similar. The average risk-based APR for this sample is 6.5%, which is very close to the uniform graduate rate charged by the Direct loan program – this again suggests that the applicant sample is representative of the distribution of risk underlying the federal portfolio.

**Maturity Choices:** Borrowers in my setting are asked to choose from a continuum of maturities from 5 to 20 years which allows them to customize their monthly payment. As borrowers extend their repayment maturity, they reduce their monthly payment but increase their total costs. There is a steep and sizable yearly payment gradient for individuals who are starting at a low maturity: individuals at a 5 year maturity pay $1300 less per year on average when they increase maturity to 6 years. However, individuals moving from a 19 to 20 year maturity pay only $200 less per year.

Given the novelty of the choice set and complexity of interest rate/monthly payment tradeoff, one might wonder if borrowers are making completely informed decisions. There are several aspects of the user interface that borrowers interact with that make this unlikely: for one, borrowers are provided with the monthly payment, APR, and total paid for the maturity they choose at many points during the refinancing process. Borrowers use a “slider” to adjust their monthly payment, and are shown how maturity, APR, and total payments change simultaneously – Figure 1 shows an example of this user interface. This means they are aware not only of the tradeoffs inherent when choosing any given maturity, but also the rate at which these tradeoffs change when they adjust maturity.⁸

The resulting distribution of maturity/monthly payment choices is shown in Figure 2. The majority of borrower are aggressive in their pay down rates and choose maturities

⁸In this way the information provided to the borrower is very similar to the information necessary for the first order condition calculations in our model: they see both the change in monthly payment associated with a maturity increase and the change in total interest paid.
Debt Amount to Refinance: $80,000

<table>
<thead>
<tr>
<th>Monthly Payment</th>
<th>$1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate</td>
<td>4.54%</td>
</tr>
<tr>
<td>Loan Term</td>
<td>8.0</td>
</tr>
<tr>
<td>Lifetime Cost</td>
<td>$95,556</td>
</tr>
</tbody>
</table>

**Figure 1: User Interface for Maturity/Monthly Payment Selection**

This figure shows the interface individuals use when selecting a loan term. The interface shows them the customized monthly payment, APR, and total paid associated with every possible term choice. This means even less financially-savvy borrowers are well-informed of the implications of their term choices.

**Figure 2: Distribution of Maturity Choices and Resulting Monthly Payments**

Both figures plot the histogram of maturity choices for the borrower sample. The left hand figure also plots the average monthly payment for the loans in each maturity bin, while the right hand figure plots the average monthly payment to monthly free cash flow (income minus housing costs) ratio for the loans in each bin. While individuals choosing shorter maturities make larger monthly payments, they tend to have higher monthly FCF. This means that across the maturity spectrum, most borrowers select into a contract that makes them pay 20-40% of their monthly FCF towards their student loan.

Below 120 months, which results in much higher monthly payments. However, there also seems to be significant sorting into maturity based on current monthly income and expenses. The right-hand figure plots monthly payment as a percentage of monthly income minus fixed expenses. This is relatively constant across maturities, ranging from 20-40%.

While the interest rate/monthly payment tradeoff is salient to borrowers, the distribution of maturity choices suggests that some borrowers may be using heuristics, like rule-of-thumb accounting, when making maturity choices. For example, the distribution of chosen monthly payments has distinct spikes at “round” monthly payments, like $500,
$1000, or $1500, while the distribution of maturity choices (see Figure 2) shows spikes at 90, 120, and 180 months. While these behavioral borrowers are in the minority, it is important to acknowledge that a rational model of financial decision making does not apply to all households. Furthermore, those with specific maturity or monthly payment targets will be less sensitive to interest rate changes, and this would bias elasticity estimates downward.

2.2.2 Price Variation:

The maturity choices described above show how borrowers select into various repayment contracts given a set of interest rates. However, they reveal little about how borrowers’ repayment decisions would change if faced with new set of interest rates. Measuring this elasticity, a central goal of this paper, requires variation in interest rates.

There are two main types of interest rate variation in my dataset: risk-based and within-risk. Using risk-based variation to identify the elasticity of maturity with respect to interest rates is potentially misleading. Individuals with different risk scores may also differ on unobservable dimensions (like expectations about future income growth or volatility) that will impact their maturity choices. Ideally one would instead use price variation that is orthogonal to all borrower characteristics, including risk score. I refer to this exogenous variation as "within-risk" variation.9

I use 10 small within-risk score price changes that were conducted at a firm-wide level, and were unrelated to the characteristics of any given borrower. The price changes were conducted primarily to gather quasi-experimental evidence for the firm on maturity choice and application volume elasticities with respect to interest rates. The price changes occurred over time, not simultaneously for different groups of borrowers, and at a frequency of once to twice a month.10 While not all price regimes lasted the same amount of time or effected the same number of borrowers, on average they each impacted 1,100 borrowers. Borrowers were not aware of the timing of these price changes, and therefore could not respond by adjusting when they refinanced.

9 Figure 12 in the Appendix provides a graphical explanation of why within-risk score price variation is necessary for identification, and how it is used.

10 This frequency helps alleviate concerns about significant changes in the composition of customers over time (a period of rapid growth), but there are still changes in the observable characteristics of the population over the full set of price changes, addressed below.
2.3 Reduced Form Maturity Elasticity wrt. Interest Rates

To estimate a reduced form elasticity, I regress each borrower’s maturity choice on observable characteristics and the interest rates that they were offered. To represent the interest rate an individual was offered, I use the 60 and 120 month fixed rate APR from the price schedule assigned to risk type \( p_i \). As discussed above, there was variation in the schedules offered to individuals of the same risk type over time. This regression tests whether observably similar individuals who were offered a higher level of interest rates made different maturity decisions.

I regress maturity choice in months \((T_i)\) on these APR variables as well as a dummied risk score variable, the offered variable 120 month interest rate, FICO score, log loan amount, monthly free cash flow, age (dummied), education level (dummied), mortgage number, and number of dependents. This specification includes dummies for risk score, and therefore uses only the remaining within-risk price variation to identify the price elasticity.

<table>
<thead>
<tr>
<th></th>
<th>Coefficients ( \frac{dy}{dx} )</th>
<th>Elasticities ( \frac{dy}{dx} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed 60</td>
<td>-4.304</td>
<td>-0.196</td>
</tr>
<tr>
<td></td>
<td>(3.309)</td>
<td>(0.151)</td>
</tr>
<tr>
<td>Fixed 120</td>
<td>-15.75**</td>
<td>-0.836**</td>
</tr>
<tr>
<td></td>
<td>(5.146)</td>
<td>(0.273)</td>
</tr>
<tr>
<td>FICO</td>
<td>-0.0570***</td>
<td>-0.440***</td>
</tr>
<tr>
<td></td>
<td>(0.0147)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>ln(amt)</td>
<td>24.65***</td>
<td>0.243***</td>
</tr>
<tr>
<td></td>
<td>(0.579)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>Monthly FCF</td>
<td>-0.00319***</td>
<td>-0.119***</td>
</tr>
<tr>
<td></td>
<td>(0.000205)</td>
<td>(0.00887)</td>
</tr>
</tbody>
</table>

Table 3: Maturity Elasticities (in Months)

Table 3 show both the raw coefficients and the coefficients transformed into elasticities. The coefficient on both the interest rate variables has a negative sign – this means that when faced with higher levels of interest rates, similar individuals decrease their loan maturity and increase their monthly payment. Specifically, a 1 percentage point increase in the 120 month APR is associated with a 15.75 month decrease in the average maturity. For the average $70,000 loan with a 108 month maturity, this means that increasing

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11The results of the regression are in the Appendix.
the average 10 year APR from 5.5% to 5.6% would increase monthly payments by approximately 1.7%, but keep total interest payments relatively constant. This elasticity suggests that the average borrower is more sensitive to the interest rate than monthly payments.

The regression also empirically confirms several other economically intuitive relationships – for example, individuals with larger loan amounts and lower monthly incomes systematically choose longer maturities. This suggests that borrowers are making proportional, not absolute, monthly payment decisions with regards to their income level. Individuals with better credit scores choose shorter maturities, even conditional on price, income, and debt amount. This may be driven by the fact that they face fewer credit constraints on other borrowing margins, or are more aggressive in the pay down of their debt.

2.3.1 Sample Selection Over Price Regimes

Using temporal price variation presents a selection concern: while some individuals may respond to interest rate changes on the intensive margin by adjusting maturity, others may respond on the extensive margin by no longer refinancing or refinancing with a different company. If those who join or leave the population after a price change have systematically higher or lower maturity preferences, then this extensive margin response will bias our intensive margin estimates.

One empirical way to gauge the extent of extensive margin responses is to test whether changes in observable borrower characteristics over price regimes are correlated with the exogenous variation in APR. If the composition of observable characteristics is predicted by the price changes, then we would be worried that there may also be selection on unobservables. Table 9 tests whether four important observable characteristics, income, debt, FICO, and savings, are predicted by the price regime shifts. These insignificant results show that price changes did not cause any differential attrition across observable characteristics: while characteristics like income and FICO did vary over price regimes, this variation was not correlated with the price level. Figure 16 graphically shows the lack of correlation between observable characteristics like debt and income and the price shifts. I also predict individuals' maturity choices, \( \hat{T}_i \), using all observable characteristics other than APR, and test whether this variable is predicted by the price regime shifts;
again, these results are insignificant and not large enough to explain the price elasticity estimated in the main specification. This makes sense since the highly competitive nature of the refinancing market, and growth of risk based pricing, means that even “high” risk borrowers have outside options that are close to their quoted price. It is helpful to note that the price changes were not monotonic: interest rates both increased and decreased over time, and therefore will not be confounded by other monotonic trends occurring over time like growth of the company.

3 Loan Repayment Model

In this section I outline a simple model which describes how borrowers make maturity and refinancing choices as they begin to repay their loans. One goal of the borrower model is to be able to interpret the reduced form maturity elasticity in terms of the intertemporal elasticity of substitution.

Borrowers entering repayment have incurred a fixed amount of student debt while attending school, $D$, are now finished with school, and are beginning repayment. In this two-stage model, borrowers choose a repayment maturity, $T$, to maximize their present discounted stream of expected future utility, and whether to refinance their federal loan into the private sector. Two main things distinguish the public and private repayment options: risk-based pricing and maturity-based pricing. The private sector offers interest rates, $r(T, p)$, that are increasing in a borrower’s observed risk $p$ and chosen maturity $T$. The government offers a single price for all risk types and maturities, $g$. Monthly and total payments could be lower or higher for a given individual in the private vs. public sector - this depends on their risk type and maturity preference.

All borrowers have the same per-period CRRA utility function $u(c) = \frac{c^{1-\gamma}}{(1-\gamma)}$, and discount factor, $\rho$. While borrowers can control their payment level, they cannot control

---

12I model all borrower decisions conditional on debt, schooling, and educational choices, which are made at an earlier period before repayment begins. This equates to the assumption that these decisions are fixed and not impacted by the level of interest rates or ability to refinance debt. This assumption is valid for the population of student borrowers who have already made their loan principal decisions and are yet to make repayment choices (i.e. those currently in school or beginning repayment) – however, it does not apply to individuals who have yet to make borrowing decisions (prospective borrowers who have yet to start school). In a full equilibrium analysis, the level of interest rates and refinancing options could also impact choices like loan principal.
their variable, growing income $\tilde{w}$, which I parametrize using a unit root process:

$$
\ln(\tilde{w}_t) = \ln(\tilde{w}_{t-1}) + u_t
$$

$$
u_t \sim N(g, \sigma^2)
$$

where $g$ is a yearly growth rate and $\sigma^2$ is income variance.

### 3.1 Step 1: Maturity Demand

Borrowers of risk type $p$ and debt amount $D$ choose a maturity $T$ to solve:

$$
\max_T \mathbb{E}\left[ \sum_{t=1}^T \beta^t u(w_t - d(T, r)) + \sum_{t=T+1}^Q \beta^t u(w_t) \right]
$$

where $Q$ is the individuals’ maximum age (i.e. finite), $w(t)$ is income plus the maximum debt payment an individual can make$^{14}$, and $d(T, r)$ is the yearly payment associated with maturity $T$. Here consumption is defined as income minus the student debt payment $d(T, r)$ – this implies that individuals are “hand-to-mouth” and not smoothing consumption through other means of borrowing or saving.$^{15}$ It also means that changes in maturity translate directly into changes in consumption by impacting the size of the yearly payment, $d(T, r)$.

The debt payment, $d(T, r)$, that individuals make each period is a function of their total debt amount $D$, their chosen maturity, $T$, and the (potentially risk, maturity-specific) interest rate schedule that they are offered, $r(T, p)$:

$$
d(T, r) = D \ast \frac{r(T, p)}{(1 - (1 + r(T, p))^{-T})}
$$

As borrowers extend maturity, each periods’ payments become lower ($\frac{dd(T, r)}{dT} < 0$), but they pay more over the life of the loan.

$^{13}$In the public sector the interest rate $r(T, p)$ is replaced with $g$

$^{14}$It is important to define $w(t)$ as yearly income plus the maximum debt payment an individual can make: $w(t) = \tilde{w}(t) + d_{\text{MAX}}$. This prevents $w(t) - d(T, r)$ from ever being $\leq 0$ and causing infinite marginal utility, even in the case of a very small draw of $\tilde{w}$. Another way of dealing with very negative income shocks is to define a minimum consumption threshold, below which an individual will no longer make a debt payment. I discuss and model this approach in the appendix.

$^{15}$I address this assumption empirically in the robustness section.
Solving the maximization problem results in the first order condition:

$$0 = -E\left[\sum_{t=0}^{T} \beta^t \frac{\partial d(t, r)}{\partial T} \left( w_t - d(t, r) \right) + \beta^{T+1} u(w(T+1) - d(T, r)) - \beta^{T+1} u(w(T+1)) \right]$$

This condition says that at the optimal loan maturity, the sum of marginal utility gained from a slightly lower monthly payment (from a slightly longer term) is equal to the utility lost from paying additional interest for an extra year.

3.1.1 The Influence of Interest Rates on Maturity Choice:

The first order condition captures how maturity choices, and therefore utility levels, change under various price regimes. All else constant, as the level of interest rates increases, individuals must decrease maturity to maintain the optimality condition. The exact formula for the response $d^{r*}_{dr}$ does not have an analytical solution, but Figure 17 in the Appendix shows how optimal term choices, simulated according to this model, vary with the level of interest rates – as interest rates increase, the optimal maturity choice decreases. It also plots this relationship for two calibrated values of the intertemporal elasticity of substitution (IES), 1.25 and .3. As the IES increases, individuals become more interest rate sensitive. This leads to a lower optimal maturity choice at any given interest rate, and also to a steeper relationship between $T^*$ and $r$.

3.1.2 The Influence of Non-Interest Rate Factors on Maturity Choice:

The other non-interest rate factors in our model that influence maturity demand and interact with the interest rate elasticity are: income level, debt level, income growth and volatility, and the intertemporal elasticity of substitution $\frac{1}{\gamma}$. Due to concave utility, individuals who are low income or high debt gain more marginal utility from decreasing yearly payments, and thus have a higher willingness to pay for long maturities. Individuals who expect income to grow in the future will also prefer a long maturity, since it acts as a means to transfer consumption from the future to the present. Individuals with higher income variability have both higher and less elastic maturity demand due to the fact that longer loans help to smooth consumption across a more variable income profile.

Note that the income-related factors that drive demand (income levels, growth, and volatility) are potentially correlated with risk score $p_i$. Therefore, the same variables that
increase demand for maturity on the borrower’s side will also increase interest rates on
the supply side. This means that even when faced with higher risk-based prices, high
risk borrowers may choose longer loans. This is in line with our reduced form evidence,
which showed that, all else constant, borrowers with lower FICO scores and risk scores
demanded longer maturities.

The first order condition also helps us understand how the intertemporal elasticity
of substitution (\( \frac{1}{\gamma} \)) influences demand. As \( \gamma \) increases, an individual will prefer a longer
maturity holding all else constant (see figure 17 for a comparison of maturity demand with
a high and low IES). Intuitively, this is because an individual with concave utility will
prefer to smooth consumption by lowering yearly payments, even if it means paying more
interest overall. A high level of \( \gamma \) (ie a low intertemporal elasticity of substitution) also
means that the term choices of individuals will be less responsive to price changes. Thus
\( \gamma \) is essential for understanding how a borrower’s decisions, and utility, would respond to
changes in price, the central goal of this study.

3.2 Step 2: Refinancing Choice

Borrowers also decide whether or not to refinance by comparing the level of utility at
the optimal maturity across sectors. For individuals on the high or low end of the risk
distribution, the risk based price differential determines whether they should refinance.
For very low risk individuals there will be a clear incentive to refinance in the private
sector, and for very high risk individuals, there will be no incentive to refinance because
government interest rates will be significantly lower than private rates. For marginal indi
dividuals who face a similar level of prices in the private and public sectors, the term based
price differential (and whether they prefer shorter or longer loans) could also determine
whether they will sort into the private sector. While the model describes the decision to
refinance as a discrete choice problem, in reality borrowers might face frictions (inertia,
search costs), value non-pecuniary repayment benefits, or have idiosyncratic preferences
that prevent them from refinancing even when they would receive lower interest rates.

3.3 Modeling Borrower Delinquency

This model ignores the impact of delinquency or default on maturity choice, focus-
ing only on the intertemporal consumption tradeoff that comes from a higher or lower
monthly payment.\textsuperscript{16} This equates to the assumption that adjusting loan maturity does not impact the probability of delinquency, despite the fact that maturity impacts the size and duration of monthly payments.

One reason for this assumption is that the dataset used to estimate the model does not observe instances of default or delinquency.\textsuperscript{17} The main model focuses solely on relationships that can estimated empirically – i.e. the response of maturity to changes in interest rates. While the dataset captures how maturity choices relate to interest rate variation, absent data on delinquency or default it is impossible to empirically link maturity choices to repayment outcomes. In the Appendix, I instead write down and analyze a model that theoretically links delinquency and maturity choice. In this model, extending maturity will decrease the probability of delinquency, \( P(T) \), in any single period by lowering the monthly payment, but may also increase the probability of delinquency over the life of the loan by extending the repayment period. Misspecification will be more or less of a problem if changes in maturity have a large impact on utility via these delinquency channels. This exercise also provides a robustness test, for various calibrated values, of the central, simplified model which I use in the estimation section. The results suggest that the estimates are not very sensitive to the exclusion of the delinquency channel – for this low risk group, the first order effect of a change in maturity seems to operate instead through the monthly payment consumption smoothing channel.

4 Estimation of Borrower Model

In this section I use the same maturity choices and within-risk price variation explored in the reduced form section to estimate the maturity demand model. This exercise estimates borrower utility using the same price variation and maturity response as in the reduced form section. While it imposes stronger assumptions on the borrower’s problem (i.e. calibrating the income process and assuming CRRA utility), it allows us to map the reduced form maturity elasticity to a parameter of economic interest, \( \gamma \), and to ultimately measure changes in consumer surplus. In this section I first discuss estimation of the first

\textsuperscript{16} Currently, borrower income levels and risk impact maturity decisions due to the fact that a low income draw minus a large debt payment will generate a very high marginal utility.

\textsuperscript{17} Note that the dataset captures only the first 6-36 months of repayment of potentially very lengthy repayment maturities (5-20 years). The absence of delinquency in these initial months does not necessarily mean that delinquency will never occur.
order condition using nonlinear least squares, then identification, results, and robustness.

Recall that individuals choose $T$ to maximize a discounted stream of utility; the resulting first order condition provides our main estimating moment. Most elements of this equation are observed, including: $T_i$, the optimal maturity choice in months; $d_i$, the monthly payment for individual $i$ at term $T_i$; $\frac{dd_i}{dT_i}$, the change in monthly payment given a one month increase in maturity from $T_i$; $r(T_i, p_i)$, the risk, maturity specific interest rate faced by individual $i$ at term $T_i$; and $w_{i0}$, individuals’ starting level of post-tax monthly income when they first make their maturity choice.

Post-tax income in future periods, $w_{it}$, is not observed. I assume log income follows a unit root process and calibrate both the growth rate and volatility.\textsuperscript{18} The calibration details are provided below – I predict the growth rate, $\hat{g}_i$, using a separate cross-sectional dataset of personal loan applicants, and calibrate income volatility, $\hat{\sigma}_i^2$, using the repayment probabilities implied by a simple lending model.

The remaining parameter left to estimate is the intertemporal elasticity of substitution, which I model as a function of observable characteristics ($X_i$) including degree type, risk score, current disposable income, student loan amount, age, FICO score, home ownership, and number of dependents:

$$\gamma_i = \gamma + X_i'\mu$$

Allowing $\gamma_i$ to vary with $X_i$ will control for changes in observable characteristics across price regimes. Following the logic outlined in the reduced form section, it is also important to include observed risk type ($p_i$) directly in the model. If $p_i$ was not included in the estimation, the model would wrongly attribute differences in maturity choices across risk type to differences in offered APR, and our estimate of $\gamma$ would suffer from omitted variable bias.

In the main specification, I use a certainty equivalence approach to write the first order condition as a closed form analytical expression. Specifically I rewrite the expected

\textsuperscript{18}In the robustness checks I relax this assumption and try other specifications.
marginal utility as the marginal utility of a certainty equivalent given by:

$$E[(w_{i0} * e^{\bar{g}_i}) * e^{\sum_t u_{it}} - d_i]^{-\gamma_i} = (w_{i0} * e^{\bar{g}_i}) * e^{\pi_{it}} - d_i]^{-\gamma_i}$$

where $\pi_{it}$ is the certain amount an individual would have to be given in that period to make the certainty equivalent equal to the expected marginal utility. As income volatility, $\gamma_i$, and the debt to income ratio $(w_{i0} * e^{\bar{g}_i} - d_i)$ ratio increase, the certainty equivalent becomes more negative. Specifically:

$$\pi_{it} = \frac{1}{2} * t * \sigma_i^2 \left[ 1 - (1 + \gamma) \right] \frac{w_{i0} * e^{\bar{g}_i}}{w_{i0} * e^{\bar{g}_i} - d_i} \quad \text{for } t < T+1$$

$$\pi_{it} = \frac{1}{2} * t * \sigma_i^2 (-\gamma_i) \quad \text{for } t \geq T+1$$

I observe individuals making a maturity choice on a monthly level. I approximate $\frac{\partial d}{\partial T}$ as the change in monthly payment given a month change in maturity. With these changes, the analytical estimating moment becomes:

$$h_i(\theta) = \sum_{t=1}^{T} \beta T \frac{\partial d}{\partial T} (w_{i0} * e^{\bar{g}_i}) * e^{\pi_{it}} - d_i]^{-\gamma_i} - \beta^{T+1} (-d_i)(w_{i0} * e^{(T+1)\hat{g}_i} * e^{\pi_{(T+1)}})^{-\gamma_i}$$

This makes the first order condition a nonlinear function of observable variables, $(r(T_i, p_i), T_i, w_{i0}, D_i, X_i, \hat{g}_i, \sigma_i^2)$, and unobservable parameters, $\theta = \{\gamma, \mu\}$, that I estimate using nonlinear least squares. I calibrate a yearly discount rate of .98.

### 4.1 Calibration of Income Volatility and Growth Rates

Estimation of the borrower problem requires calibration of income growth and volatility. Here I explain how I derive implied income risk from a simple lending model and estimate income growth rates.

**Lending Model:** I assume that lenders in this sector are perfectly competitive, and therefore set interest rates such that they are indifferent between lending to a risky borrower at interest rate $r(p, T)$, and lending to a risk-free borrower at interest rate $i(T)$.

---

19 For the exact derivation of $\pi_{it}$ see Appendix
20 Namely, $\frac{\partial d}{\partial T} = d(T+1) - d(T)$
21 To make this condition empirically tractable, I replace the second term, $u(w_{(T+1)} - d(T, r)) - u(w(T+1))$, with the mid-point approximation $-d(T, r) * u'(w(T+1)) - \frac{d(T,r)}{2}$. 
I assume that the risk-free rate \( i(T) \) also incorporates the fixed costs that the company must incur when lending (i.e. origination costs and cost of capital), and therefore the only difference between \( i(T) \) and \( r(p,T) \) is the risk premium.

In the Appendix, I write down a model in which borrowers will be delinquent on a loan in a given period if income falls below a certain threshold – i.e. \( w(t) - d(T) < x \).

This means that the probability that a borrower is delinquent in a given period is:

\[
P(T) = \frac{\Phi(ln(x + d(T)) - ln(w_0))}{\sigma_i} \tag{1}
\]

Lenders have a recovery rate of \( \alpha \) if a payment is delinquent. Given the delinquency probability and recovery rate, a lender’s expected stream of payments on a loan of maturity \( T \) to risk type \( p \) is:

\[
T \ast \left( (1 - P(T)) \ast D \ast \frac{r(T,p)}{(1 - (1 + r(T,p))^{-T})} + \alpha \ast P(T) \ast D \ast \frac{r(T,p)}{(1 - (1 + r(T,p))^{-T})} \right)
\]

while the expected stream of payments on a risk-free loan of maturity \( T \) is:

\[
T \ast \left( D \ast \frac{i(T)}{(1 - (1 + i(T))^{-T})} \right)
\]

Lenders set \( r(p,T) \) to satisfy the indifference condition:

\[
P(T) = \left( \frac{i(T)}{(1 - (1 + i(T))^{-T})} \ast \frac{(1 - (1 + r(T,p))^{-T})}{r(T,p)} - 1 \right) \ast \frac{1}{(\alpha - 1)} \tag{2}
\]

I observe everything on the right-hand side of this equation, which will allow me to calculate empirical delinquency probabilities for every individual in the dataset, which I call \( \hat{P}(T) \). Combining \( \hat{P}(T) \) with equation (1) allows me to then calculate individual specific \( \hat{\sigma}_i \):

\[
\hat{\sigma}_i = \frac{\Phi(ln(x + d(T)) - ln(w_0))}{P(T)}
\]

**Calibration of Income Volatility:** I directly observe the empirical values of \( r(T,p) \) and \( T \) for each borrower, and infer values for \( \alpha \) and \( i(T) \) from the firms’ cost accounting documentation. Table 10 lists the summary statistics for these variables. The calibrated recovery rate \( \alpha = .15 \) reflects the model’s per-payment recovery assumption – i.e. if a
borrower misses a single payment, the lender will recover α of that period’s payment, and then continue to collect the remaining loan capital at the full rate. In reality, the lender often charges-off the entire loan to a collections agency, which means they may have a higher average recovery rate that applies to the entire loan principal, but a very low recovery rate in any single period (the model’s α).

The "risk-free" rate of lending, i(T), captures various costs of lending for the firm at that maturity, including the cost of capital, the cost of customer acquisition, and the cost of servicing the loan. In fact, the only cost not included in this measure is the expected cost of non-repayment (i.e. delinquency). Therefore, i(T) is the same for borrowers of all risk types, but does vary with the maturity of the loan. In contrast, r(T) varies with both the risk type of the borrower and the maturity of the loan. Equation 2 and these values produce estimates of \( \hat{P}(T) \) in the range of .003 to .15 (see Figure 13 for a histogram of the calculated probabilities). For context, historical yearly delinquency and default rates for graduate students in the government’s Direct Loan portfolio range from 3-7%.

I next transition to the borrower problem, and calculate the individual-specific income volatility that would generate the estimated values of \( \hat{P}(T) \). This relies on Equation 1, which, under the assumption that income is log-normally distributed, maps \( \hat{P}(T) \) to \( \hat{\sigma}_i \). For these calculations, I use post-tax income as an empirical measure of \( w_0 \), and individuals’ verified housing expenditures as a proxy for \( x \), the minimum consumption threshold. This measure of \( x \) assumes that individuals will first pay their rent or mortgage before making their student debt payment. As \( x \) becomes larger relative to \( w_0 \), \( \sigma_i \) will become smaller: a smaller shock to income is needed to push an individual into delinquency.

The resulting income variance estimates are described in the final row of table 10. The median value of \( \sigma_i \) is .29, off a median value of \( ln(w) \) of 11.16. To make these numbers easier to interpret, figure 14 converts from logs to levels: for each level of post-tax yearly income, it plots the one and two standard deviation range calculated using the median value of \( \sigma_i \) for individuals in that income category. This graph says, for instance, that the median individual initially making $50,000 has a 68% chance of making anywhere between $40,000 and $82,000 in each of the following years. Figure 4 also compares the calibrated values of \( \sigma_i \) to those observed in a cross-section of earnings of individuals of the same age and degree type – they are of similar magnitude.

**Calibration of Income Growth Rates:** While one would ideally use panel data to
directly observe the income growth of my borrowers, the long time horizon of the debt contracts (up to 20 years) is a limiting factor. I instead use a cross section of observationally similar individuals at various ages to estimate a pseudo age-income profile.

The dataset I use to estimate these cross-sectional profiles contains individuals who are similar to my refinancing applicants in many important respects (high income, high FICO, mainly graduate degree recipients), but who are applying instead for small personal loans rather than applying to refinance student debt. This distinction is important when estimating cross-sectional age profiles – if I instead used a cross-section from the student loan borrower population, one might worry that individuals refinancing student debt at age 40 have very different income trajectories than those refinancing at age 30. Here the worry is that individuals borrowing small amounts ($5,000 - $15,000) at different ages have fundamentally different earning trajectories. While this selection concern is valid, one must weigh it against the fact that this population is similar to my borrowers in many unique respects that would be difficult to find and match to in a survey dataset. For example, my population is refinancing with a new internet-based bank, which makes them potentially different than a population that uses only traditional banks. Furthermore, because my sample has a high socioeconomic status, they make up only a small percentage of most representative survey samples.

This dataset contains approximately 250,000 borrowers who range in age from 20 to 50. Figure 3 gives a sense of what the cross-sectional income trajectories look like for individuals in this sample – it plots monthly post-tax income after separating individuals into 4 degree levels: associates, bachelors, masters, and professional. It also compares the growth rates and levels to those found in the CPS – in large part the two datasets seem to capture similar earnings trajectories, at least at the level of degree type.

Using this sample, I estimate degree-specific growth rates that match the log-income parametrization of my model with the following regression:

\[
\ln(w_i) = \beta_0 + \beta_1 \cdot age_i + \gamma_0 \cdot degree_i + \gamma_1 \cdot age_i \cdot degree_i + e_i
\]

Here \(\ln(w_i)\) is log post-tax yearly income, and \(degree_i\) are dummies indicating highest degree level. The coefficient \(\hat{\beta}_1\) estimates the average yearly growth rate of log income over the life-cycle, while \(\hat{\gamma}_1\) allows this growth rate to deviate by degree type. The estimated growth rate (see Figure 19) for any borrower in my student loan dataset is thus given by
\( g_i = (\hat{\beta}_1 + \hat{\gamma}_1).^{22} \)

\[
\begin{align*}
&\text{Personal Loan Data} \\
&\text{CPS Data}
\end{align*}
\]

(a) Personal Loan Dataset (used for estimation) \hspace{1cm} (b) CPS Dataset

Here I plot a comparison of monthly post-tax income for individuals with different degree types and occupations within the dataset I use for estimation and the CPS. The personal loan data that I use for estimation seems to reflect a similar growth pattern to that found in the CPS, in particular for Associates, Bachelors, and Master degree recipients. Professional degree holders in the CPS data have a slightly higher level and steeper growth rate of monthly post tax income.

Finally, we can use this personal loan dataset to provide a second estimate of income volatility to compare with our calibrated volatility estimates. In Panel (a) of Figure 4, I plot the standard deviation of observed yearly log income for age-degree specific groups using the personal loan dataset. In Panel (b), I plot the standard deviation of yearly log income for the same groups calculated from the risk-based interest rates. While income variance is of a similar magnitude using either method, it seems to increase with age when using the cross-sectional dataset and decrease with age when using the calibration exercise. This suggests that income patterns diverge across individuals with time, whereas within person income becomes less volatile with age.

4.2 Empirical Identification:

The identification intuition for our structural model remains very similar to that of the reduced form section: there, maturity choice was expressed as a linear function of observables, risk type, and interest rate \((X_i, p_i, \text{ and } r(T,p)_i)\), with the interest rate coefficient \((\beta)\) identified using the series of price changes that were orthogonal to \(p_i\). In the structural estimation, \(T_i\) is instead expressed as an implicit non-linear function of \(X_i\),

---

22While we would like to estimate \(g_i\) that vary by many characteristics, from number of dependents to FICO score, the use of cross-sectional data only allows for comparison on time-invariant characteristics like degree type, or occupation. Gender would be another relevant characteristic to include in this regression, but unfortunately it is not recorded in the dataset.
Figure 4: Std. Deviation of Log Yearly Post-Tax Income, by Age and Degree

In Panel (a), I plot the standard deviation of observed yearly log income for age-degree specific groups. In Panel (b), I plot the calibrated standard deviation of yearly log income for the same groups. While income variance is in the same range for both methods, it seems to increase with age when using the cross-sectional dataset and decrease with age for the calibration exercise.

\( p_i \), and \( r(T, p_i) \), derived from the borrower’s first order condition, and the price changes now serve to identify \( \gamma \).

In both models, the price changes provide moment conditions in which observationally identical individuals (in terms of both \( X_i \) and \( p_i \)) face a different set of interest rates (\( r \) vs \( r' \)) and make potentially different maturity decisions. The parameters, \( \beta \) and \( \gamma \), must rationalize how changes in maturity decisions respond to changes in interest rates. They are therefore estimated using shifts in the maturity distribution over price regimes, and not the maturity distribution itself. Using this price variation requires the assumption that conditional on \( X_i \) and \( p_i \), any unobserved characteristics of borrowers across price regimes are uncorrelated with their maturity choices.

One key difference between the two specifications is that income growth and volatility are directly modeled in the structural section. In the reduced form analysis, income growth and volatility are one of many sources of unobservable heterogeneity that might impact maturity choice – they are left in the error term, and assumed to be orthogonal to the price changes. In the structural analysis, income growth and volatility become a source of observable heterogeneity. By modeling the income path over time and over states of nature, we can translate changes in maturity into changes in consumption. While this puts more structure on the borrowers problem, it also allows us to interpret the reduced form maturity elasticity as a more economically relevant parameter, the IES.

In the model, \( \gamma \) plays the role of both the IES and the risk aversion parameter.
ically, the parameter is identified from a consumption smoothing decision – how changes in monthly payment respond to changes in interest rates – and thus should be interpreted as the IES. In the counterfactual, changes in welfare also come from intertemporal changes in consumption, not changes in income risk.

A final concern is to correctly attribute what changes in term choice across price regimes comes from actual changes in interest rate, and what portion comes from changes in sample composition. Again, modeling $\gamma$ as a function of $X_i$ and $p_i$ will control for the impact of observable heterogeneity across price regimes on maturity choice.

4.3 Results

The results from the structural estimation are shown in Table 4. The first column estimates come from our preferred specification, which models $\gamma$ as a function of observable characteristics, and the remaining columns report results from specifications with alternative assumptions, discussed in more detail below.

The average estimate of $\gamma$ in the primary, and in all specifications, falls on the moderate to low end of estimates in the existing literature. It translates into an IES of .55, and most recent micro estimates have found a IES from .2-.6. This value implies that on average there is a sizable consumption response to changes in interest rates. The small estimate of $\gamma$ is not surprising in light of our sample, setting, and earlier results. It is reflected in both the distribution of term choices, in which one quarter of borrowers choose a maturity below 6 years, and the reduced form results, which found a relatively large maturity elasticity.

Individuals actively refinancing their loans are also likely more cognizant of the interest rate tradeoffs they are making then individuals in studies that examine credit card use or saving rates. The online interface also explains how interest rates and debt maturity interact, potentially making my sample more informed than those studied in a traditional loan setting. These are complex calculations that the borrower may not make independently, and make the total interest/monthly payment tradeoff extremely salient.

It is also possible that the type of debt studied here could also have a unique psychological impact on estimates of $\gamma$. The amount of student debt a borrower has is more often determined by “necessity” (due to the level of tuition or financial aid available at the individual’s school), rather than choice and thus be perceived as more burdensome.
and unwanted. Therefore borrowers might treat their student loans differently than other forms of debt or savings, and want to pay it off more quickly. This results underscore the importance of considering several models of consumer behavior when analyzing saving and borrowing decisions - while our lifecycle model of repayment rationalizes these choices under the assumption of full information and rational expectations, there may in fact be behavioral tendencies, for example debt aversion or rule of thumb accounting, that are driving some portion of individuals’ behavior.

The main specification estimates $\gamma$ as a function of observables, returning a distribution of predicted values from 1.2 to 2.6. As explained in the identification section, this helps isolate the exogenous interest changes as the main source of identifying variation for the intertemporal elasticity of substitution, but it also allows us to compare the IES across different characteristics. There is a lower elasticity amongst older individuals, those with larger families, and those with lower credit scores. Perhaps more surprising is that individuals with higher income, lower amounts of student debt, and lower risk scores have a lower IES, which seems to contradict the reduced form results. However, heterogeneity in the IES estimates should not necessarily map 1-to-1 to the reduced form findings: the structural model allows for much more complexity than our OLS analysis, as it captures, for example, the curvature of the monthly payment tradeoff at different points in the maturity distribution, the full interest rate schedule, or the role of expected income growth. A high income individual may be making a much smaller consumption tradeoff when they reduce maturity by one month than a low income individual making the same decision. Therefore, even though low risk or high income individuals may have a higher reduced form maturity elasticity, when these additional non-linear factors are considered, they actually have a lower IES.

4.4 Robustness Analyses:

In this section I test the robustness of my modeling assumptions in several ways: first, I test how sensitive estimates of $\gamma$ are to the inclusion or exclusion of delinquency in the borrower’s problem. Second, I examine whether borrowers are adjusting on other financial margins, to see if changes in maturity truly represent changes in consumption. Finally, I test the modeling assumption that borrowers are making a single, not recurrent, maturity choice by looking at prepayment rates in the data.
<table>
<thead>
<tr>
<th>Coefficients</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>γ - Constant</strong></td>
<td>1.704</td>
<td>1.416</td>
<td>5.900</td>
<td>8.369</td>
</tr>
<tr>
<td></td>
<td>(0.269)</td>
<td>(0.01)</td>
<td>(1.771)</td>
<td>(3.122)</td>
</tr>
<tr>
<td><strong>γ - log(Income)</strong></td>
<td>0.203</td>
<td>2.356</td>
<td>3.269</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.130)</td>
<td>(0.247)</td>
<td></td>
</tr>
<tr>
<td><strong>γ - log(Debt)</strong></td>
<td>-0.234</td>
<td>-2.361</td>
<td>-3.434</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.064)</td>
<td>(0.147)</td>
<td></td>
</tr>
<tr>
<td><strong>γ - Home Owner</strong></td>
<td>-0.0532</td>
<td>0.1689</td>
<td>0.1484</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0126)</td>
<td>(0.0800)</td>
<td>(0.1328)</td>
<td></td>
</tr>
<tr>
<td><strong>γ - # Dependents</strong></td>
<td>3.76E-02</td>
<td>2.56E-01</td>
<td>3.50E-01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8.60E-03)</td>
<td>(4.83E-02)</td>
<td>(8.44E-02)</td>
<td></td>
</tr>
<tr>
<td><strong>γ - Age</strong></td>
<td>6.10E-03</td>
<td>2.40E-02</td>
<td>3.08E-02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.10E-03)</td>
<td>(7.15E-03)</td>
<td>(1.31E-02)</td>
<td></td>
</tr>
<tr>
<td><strong>γ - Risk Score</strong></td>
<td>0.124</td>
<td>-0.455</td>
<td>-0.464</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.047)</td>
<td>(0.081)</td>
<td></td>
</tr>
<tr>
<td><strong>γ - FICO</strong></td>
<td>-3.00E-04</td>
<td>-1.98E-03</td>
<td>-2.15E-03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.00E-04)</td>
<td>(1.14E-03)</td>
<td>(1.97E-03)</td>
<td></td>
</tr>
<tr>
<td>α60</td>
<td>2.477</td>
<td>4.219</td>
<td>4.691</td>
<td></td>
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<tr>
<td></td>
<td>(0.315)</td>
<td>(0.38)</td>
<td>(0.349)</td>
<td>(0.424)</td>
</tr>
<tr>
<td>α240</td>
<td>-71.695</td>
<td>-72.00</td>
<td>-73.043</td>
<td>-84.091</td>
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<tr>
<td></td>
<td>(1.992)</td>
<td>(2.06)</td>
<td>(2.149)</td>
<td>(2.400)</td>
</tr>
<tr>
<td>γ</td>
<td>1.801</td>
<td>1.416</td>
<td>4.688</td>
<td>5.892</td>
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<tr>
<td>σγ</td>
<td>0.242</td>
<td>1.673</td>
<td>2.407</td>
<td></td>
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<tr>
<td>N</td>
<td>11585</td>
<td>11585</td>
<td>11585</td>
<td></td>
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</tbody>
</table>

Standard errors in parentheses. This table presents results from the non-linear least squares estimates of the borrower’s maturity choice model. Income is defined as yearly post-tax income. Debt is the amount of student loan debt the individual is refinancing. Home Owner is a dummy indicating whether an individual owns a home. Risk Score is the firm specific score that is used as the basis for risk based prices; a higher score indicates lower risk. α(x) are dummy variables that indicate whether an individual chose the minimum or maximum maturity to account for the truncation of the maturity choice set at 5 and 20 years.

Column (1) presents the main specification, in which the intertemporal elasticity of substitution parameter (γ) is modeled as a function of observables; Column (2) does not allow γ to vary with observables. In both (1) and (2) the future income growth and volatility of the borrowers are calibrated using an external data source (see the appendix for details). In (3) I only calibrate income growth, and make income deterministic (not risky). This increases the estimate of γ, since without income volatility individuals must have a lower IES to rationalize the same maturity choices in the data. In (4) I remove income growth (income stays constant) and income volatility. Again, this increases the estimate of γ.
4.4.1 Impact of Delinquency on Estimates

While the estimated model abstracts away from delinquency, in the Appendix I write down a model that includes delinquency and adds additional terms to the first order condition. Here I calculate what happens to $\hat{\gamma}$ as we increase or decrease the magnitude of the additional terms.

The first order condition with delinquency included an unobservable utility “penalty”, $G$, that a borrower would face if they ever missed a payment on their loan. The value of $G$ that would provide the same estimate of $\gamma$ using either the delinquency FOC or the simplified FOC is $G = (-.1269E-04)^2$ – this penalty is equivalent to 1.4 years of utility at the starting income level, or cutting utility to 97% of the starting level for the next 50 years. Figure 5 panel 2 tests how $\gamma$ changes across a broad range of values of $G$, from a penalty that would reduce utility by 0% for the next 50 years, all the way to 70%. The value of $G$ implied by our estimate of $\hat{\gamma}$ is relatively low, suggesting our estimate may be biased downwards. But even across a wider domain of penalty values, the implied range of $\gamma$ is relatively narrow.

I also test how a range of delinquency probabilities, $P(T) = .001$ to $.3$, would impact an estimate of $\gamma$. Again, the value of $\gamma$ does not change considerably, moving from 1.6 to 2. The insensitivity of $\gamma$ to these additional terms makes some intuitive sense. For borrowers in our dataset who are relatively low risk, the derivative of the probability of delinquency with respect to maturity is small. This means that the first order impact of a change in maturity on utility comes from the change in monthly payment, which we capture in the main specification, not from the change in the probability of delinquency.

4.4.2 Contemporaneous Financial Decisions

Identification of $\gamma$ hinges on the fact that quasi-experimental variation in interest rates caused otherwise identical individuals to choose slightly different monthly payments, $d(T)$. The hand-to-mouth assumption in the model maps these changes in $d(T)$ directly to changes in consumption. In reality individuals may be saving, rather than consuming,
Figure 5: Comparison of $\gamma$ under delinquency and non-delinquency model.

this residual, or adjusting on other borrowing margins. Therefore, we want to test whether there was a differential change in the saving or borrowing rate before and after refinancing for individuals who faced slightly different interest rate schedules.

I directly observe the outstanding debt and debt payment of borrowers in my sample, which are substantial. Over 40% of borrowers have a mortgage, which on average translates into a $1,900 payment. All of the borrowers have some sort of fixed monthly payment on their credit reports: 40% of borrowers have monthly auto payments which are on average $450, 75% have credit card payments, and 90% have uncategorized installment debt.

However, the real test is not whether individuals have savings or debt in the sample, but whether they change their saving/borrowing behavior after refinancing and whether these changes are correlated with the price experiments. I can test this assumption by looking at borrowers' monthly payments before and after refinancing, and measuring whether they adjust immediately during refinancing. Table 20 in the appendix describes changes in other monthly payments (mortgages, auto loans, credit cards, etc) before vs. after refinancing for individuals who had positive monthly payments to begin with, and shows that for the vast majority of borrowers these stayed constant. This makes sense, since many of these payments are fixed installments, and it would take active work on the borrower’s part to readjust them.

While I do not observe savings rates before and after refinancing, I do observe the
level of savings of borrowers in my sample, which is low. Slightly under 40% have a formal retirement savings account – for example 25% have a 401k, with a median balance of $24,000. The number of individuals with investment holdings increases with age. Figure 21 shows that while the median borrower continues to not have substantial savings through age 60, the 75th percentile has accumulated over $80,000 by age 50. However, 90% of my borrowers are under 40 years old, and therefore even the most active savers have investment holdings that are much smaller than their student debt amount.

4.4.3 Evidence on Permanence of Term Choice:

Our model assumes that borrowers make a maturity choice in year 1 to maximize expected utility over the life of the loan. One might question whether borrowers are actually optimizing over such a long time horizon, or if they are in fact choosing a monthly payment to fit their current income level, with the intent to refinance and change term yet again in the future when their income level changes.

To address this, I look at payment patterns over time within my sample of refinancers – in other words, do any individuals keep their payment level over time constant, or do they systematically make higher or lower payments on their debt. I find that there are some extra payments in the data, but they are small and do not vary systematically over time. Figure 18 in the Appendix shows that each month borrowers pay on average 1.5% more than their regular payment, and this is driven by on average only 1% of borrowers making a extra payment each month. There is also no systematic trend in the extra payments. One might expect payments to increase with time as income increases, but here the level of extra payments stays constant over the two year period.

5 Welfare Analysis:

In this section, I use the estimated demand model to analyze how advances in private sector risk-based pricing impact the size and distribution of consumer surplus. My benchmark for these comparisons pools all borrowers (high and low risk) under a uniform interest rate, which represents how the federal loan program would operate with no private refinancing option. I measure the extent of cross-subsidization generated under uniform pricing across risk types and income levels, as well as the deadweight loss. I next
introduce a private refinancing option with varying degrees of pricing sophistication, from very coarse (FICO score) to fine grained (our risk score \( p_i \)), and measure two main effects: the net increase in consumer surplus, as low risk types refinance into lower, more efficient risk-based prices, and the increase in average cost for the federal program as low risk types select into the private market.

For these exercises, I use the sample of all refinancing applicants – individuals who received a refinancing price quote, but who did not necessarily complete the entire refinancing process. This is different from my estimation sample, which included only approved, agreed refinancers. The applicant sample is more representative of the federal loan portfolio, but the exercise requires that I extrapolate my estimates to a group with a much wider distribution of income, FICO score, and debt amount. To limit the extent of the extrapolation, I restrict the applicant sample to individuals who have a debt-to-income ratio that overlaps with the support of the refinancing sample.

5.1 Supply Side Assumptions:

In the private refinancing sector, I assume that the risk-based interest rates that firms offer \( r(T, p_i) \) are equal to the expected costs of lending to individual \( i \) over maturity \( T \). This equates to the assumption of a perfectly competitive refinancing market - i.e. if a firm charged a mark-up, another firm could enter the market and offer a slightly lower price to the same individual while still breaking even. The refinancing market displays most features of perfect competition, including rapid entry into the industry by many firms, and little product differentiation. It is very easy for consumers to price shop and compare price quotes online across refinancing firms. I also estimate very high elasticities (larger than 5) to refinance with respect to offered interest rates in the data, which suggests that price competition across refinancing firms is very high.\textsuperscript{25} Importantly, this assumption means that there will be no changes in producer surplus \( (PS = 0) \) during the counterfactual, only consumer surplus.

For the federal loan program, I use risk-based discount rates to estimate the size of each per-borrower subsidy under a uniform interest rate regime – specifically, I discount future cash flows under the uniform price regime with the risk, maturity-specific interest rates that would be assigned to that loan in the private sector.\textsuperscript{26} The risk-adjusted stream

\textsuperscript{25}See Table 6 in the Appendix for the analysis regression underlying this elasticity.
of cash flows, where monthly payments under uniform pricing are given by \( d_i(g) \) and term choice is \( T \), is:

\[
PRDV_i(g) = \frac{d_i(g)}{r(T, p_i)} \left[ 1 - \frac{1}{(1 + r(T, p_i))^T} \right]
\]

The value of the subsidy is given by the difference between the risk-adjusted present value of the loan, and the loan principal (which is equivalent to the present value of the loan without risk adjustment). The subsidy is positive high risk borrowers and negative for low risk borrowers. If the government were to conduct revenue-neutral pricing, the breakeven interest rate \( g \) would be defined by:

\[
\tilde{g} = \{ g : \sum_{i=1}^{N} D_i - \sum_{i=1}^{N} PRDV_i(g) = 0 \}
\]

5.2 Baseline: Fully Uniform Pricing

As a benchmark for my all my analyses, I assume individuals in my sample are forced into a uniform pricing scheme at the rate that is revenue neutral. Uniform pricing will generate deadweight loss as some individuals choose maturities at a price that is above or below the expected cost of providing to them - the low risk types end up choosing shorter loans than they would in a setting where they are charged the cost of providing the loan, and this distortion is large because of their elastic demand. Figure 6 illustrates how the equilibrium maturity choices of low and high risk types under uniform pricing \((T_{L,g} \text{ and } T_{H,g})\) are pushed further apart then in the efficient setting \((T_{L}^{*} \text{ and } T_{H}^{*})\). The graph also illustrates how uniform pricing will also increase equity, effectively “taxing” low risk types in order to “subsidize” high risk types.

Using our model, we can quantify both the deadweight loss and the redistribution represented here graphically. The breakeven interest rate for this sample is \( g = 6.4\% \), which is in the range of existing Federal Interest Rates for graduate students. To calculate the deadweight loss associated with the uniform interest rate, I calculate the transfer, in addition to the revenue collected, that would make each individual indifferent between a

\[26\] By using these market prices, I focus only on risk that is observable and priced in the private sector but unpriced in the public sector. I also assume that expected losses given risk type and term are the same in the private and public sector. This assumption seems reasonable given that both sector treat default and delinquency similarly.
This figure depicts the maturity choices a high and low risk borrower would make in the private sector and the public sector. There are two maturity demand curves, the lower one for the low risk borrower, and the higher one for a high risk borrower. In the private sector they face the two risk specific price schedules, \( r(p_H) \) and \( r(p_L) \), and choose terms \( T^*_H \) and \( T^*_L \), which are efficient. In the public sector, they instead both face the uniform price \( g \). The low risk type chooses a much shorter term \( T_{G,L} \) and the high risk type chooses a much longer term \( T_{G,H} \) than in the private sector.

Simply returning the revenue to the borrower is insufficient compensation, since the maturity elasticity (i.e. the substitution effect) is nonzero. On average, the per borrower DWL due to the maturity distortion is $448, or 32% of the average tax/transfer. These calculations suggest that a uniform interest rate is a relatively inefficient means of redistribution, which is unsurprising in light of the high estimated IES and maturity elasticity.

Redistribution occurs primarily over risk type, given that risk type directly determines an individual’s true “price” and therefore the size of the implicit tax or subsidy they face. On average, individuals who are low risk are taxed $1,082 under uniform pricing (relative to risk-based pricing), whereas individuals who are high risk (and thus face lower interest rates under uniform pricing) gain an average of $1,507. The redistribution achieved over a more equity-relevant variable, income, is modest. Figure 7 plots the average subsidy given to each borrower under uniform pricing over both borrower risk type and borrower income. The lowest income borrowers get a subsidy of slightly more than $1000, while the riskiest borrowers get an average subsidy of almost $3,000. This is because income is not perfectly correlated with risk type or maturity preferences (the two dimensions that differentiate costs and thus directly generate redistribution), and therefore the uniform pricing regime.

\(^{27}\)This requires predicting individual’s optimal maturity choice under each pricing scenario using the calibrated demand model, and calculating the size of the subsidy or tax they face at that maturity under the uniform rate.
rate is an imperfect instrument for achieving redistribution over income.

![Figure 7: Redistribution over Risk and Income under a Uniform Interest Rate](image)

This figure plots the average per borrower tax or transfer under a uniform interest rate policy for 20 income quantiles and 20 risk type quantiles. While the redistribution from lowest to highest risk quantile is large (over $3,000 on average per borrower), less redistribution occurs from lowest to highest income quantile.

5.3 Counterfactual I: Innovations in Risk-Based Pricing and Expansion of the Private Refinancing Market

I next analyze what happens to sorting and welfare in the market as risk-based pricing technologies advance and refinancing firms are able to price on more characteristics. Panel (a) of Figure 8 shows how innovations in risk-based pricing increase the distribution of interest rates charged in the private sector relative to a more coarse measure of borrower quality like FICO score. Here I calculate and plot the 10 year fixed interest rate each borrower would face if the firm could only price on FICO score, as well as the 10 year fixed interest rate each borrower would face at the current “frontier” of risk-based pricing.28 The graph shows that more comprehensive risk-based pricing expands the distribution of interest rates, in particular extending lower interest rates to the least risky types. The gains to considering additional characteristics are especially large for the student

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28I use the observed interest rates schedules from my dataset as empirical proxies for $r(T, p_i)$. The firm estimates borrower risk using a predictive algorithm to estimate the probability of delinquency. This algorithm produces a risk score $p_i$ for each individual based on a vector of characteristics, $X_i$. This score maps to a schedule of risk, maturity specific interest rates $r(T, p_i)$. 

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borrower population: because these borrowers are young and have less-developed FICO scores, this allows them to signal their risk type through other characteristics like degree type, savings behavior, or income.

Figure 8: Distribution of Interest Rates when Pricing on Different Observables
Panel (a) shows how innovations in risk-based pricing increase the distribution of interest rates charged in the private sector relative to a more coarse measure of borrower quality like FICO score. Here I calculate and plot the 10 year fixed interest rate each borrower would face if the firm could only price on FICO score, as well as the 10 year fixed interest rate each borrower would face if the firm could price on a more comprehensive set of variables including monthly free cash flow, assets, degree type, and occupation. Panel (b) shows how further restricting the set of variables a firm could price on, for example to school rank or degree type, would substantially reduce the spread of the price distribution.

Using these prices, we can calculate changes in borrower surplus as low risk individuals refinance out of the public sector to take advantage of lower rates. In this initial analysis I assume all individuals who would benefit from refinancing do so. Table 5, Column 1, displays the average gain from risk-based pricing, defined as the change in present discounted cash flows, for individuals who refinance. By leaving the uniform regime, these consumers gain on average $1082.52. Using comprehensive risk-based pricing, rather than FICO-based pricing, increases these gains substantially by $341 per borrower.

From the government’s perspective, when these low-risk individuals leave the break-even program to refinance, the average risk of the remaining pool increases. Under the assumption that the government maintains the same interest rate even when low risk types exit, I calculate the average per borrower subsidy for the remaining set of borrowers in column 2. The exit of low risk types under full risk-based pricing increases the subsidy from $0 to $1,507. While innovations in risk-based pricing increased the gains for individuals who refinanced relative to FICO-based pricing, the two pricing
Table 5: Impact of Pricing and Refinancing on Surplus and Revenue

<table>
<thead>
<tr>
<th>Average Changes in Surplus under Different Pricing Schemes</th>
<th>Average Tax*</th>
<th>Average Subsidy**</th>
<th>Average DWL***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Pooling, Break-even g</td>
<td>1082.52</td>
<td>1507.48</td>
<td>448.19</td>
</tr>
<tr>
<td>Complete FICO-based pricing</td>
<td>580.99</td>
<td>745.80</td>
<td>13.06</td>
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<tr>
<td>Complete Risk-based pricing</td>
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</tr>
</tbody>
</table>

Average Changes in Surplus as Individuals Refinance Out of Break-even Pool

<table>
<thead>
<tr>
<th>Refinance into FICO-based Prices</th>
<th>Avg. Δ PRDV, for Refinancers †</th>
<th>Avg. Δ Subsidy††</th>
<th>Avg. Δ DWL†††</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>741.20</td>
<td>1259.80</td>
<td>-174.29</td>
</tr>
<tr>
<td>Refinance into Full Risk-based Prices</td>
<td>1082.52</td>
<td>1507.48</td>
<td>-221.87</td>
</tr>
</tbody>
</table>

* Average Tax defined as the average negative change in present risk discounted value relative to the risk-based price regime. For government pooling this change is defined as, $D_i - PRDV_i(g)$. For FICO-based pricing this is defined as $D_i - PRDV_i(r(FICO_i))$. The present value is risk adjusted using the risk-based interest rates $r(p_i)$, which means that $PRDV_i(r(p_i)) = D_i$.

** Average Subsidy defined as the average positive change in present risk discounted value relative to the risk-based price regime.

*** DWL is calculated as the transfer, in addition to the tax or subsidy collected, that would make each individual indifferent between $g$ or $r(FICO_i)$ and $r(p_i)$.

† Average Δ PRDV for refinancers is defined as $PRDV_i(r(FICO_i)) - PRDV_i(g)$ for individuals refinancing under a FICO based regime. For individuals refinancing under a fully risk-based pricing regime, $Δ PRDV_i = PRDV_i(r(p_i)) - PRDV_i(g) - D_i - PRDV_i(g) = average tax under full pooling.

†† Avg. Δ Subsidy is defined as the new average subsidy the government will be providing under their original breakeven interest rate for individuals who do not refinance into the private sector

††† $Δ DWL$ is defined as $DWL(r) - DWL(g)$. There will be no change in DWL for individuals who do not refinance – $DWL(g) - DWL(g) = 0$. There will be a decrease in DWL for individuals who refinance, but the change will be smaller for individuals refinancing into FICO based prices – $|DWL(r(FICO_i)) - DWL(g)| < |DWL(r(p_i)) - DWL(g)| = DWL(g)$.

Schemes generate smaller changes in the new subsidy, only a $247 value. This is because innovations in risk-based pricing do not increase the extensive number of refinancers, but rather extend lower interest rates to individuals who would have already benefited from FICO-based pricing and exited the public pool.

Finally, we can calculate the decrease in deadweight loss that occurs as accurate risk based prices correct the maturity distortion for the group of borrowers who refinance. While FICO-based prices are more accurate than an entirely uniform interest rate, they still generate some (very small) maturity distortions and deadweight loss.

5.4 Counterfactual II: Transitioning to a Net Subsidy

The previous counterfactual showed that as risk-based pricing advances, low risk types will exit the public sector and the originally break-even rate will become an effective subsidy for the remaining borrowers. It is unclear whether the original value of $g$ minimizes the cost of this subsidy. In fact, our model highlights how policy makers must consider behavioral responses on two budget relevant margins when setting $g$: maturity choice and refinancing decisions. These responses change the costs associated with charging any given interest rate $g$.

The graphs in Figure 9 analyze how subsidy costs vary for the government with $g$, allowing for various behavioral responses by borrowers. The horizontal axis represents the different values of a uniform interest rate $g$ that the government could charge on.
federal loans. On the vertical axis I plot the amount that the government will *raise* and *spend* on the federal loan portfolio for any value of $g$, accounting for heterogeneous risk types, maturity choices, and refinancing choices. Therefore, the net interest rate tax or subsidy provided by the government at any given value of $g$ can be found by tracing out the vertical distance between the dotted revenue line and the solid cost line at that point. The value of $g$ that will allow the government to break even on the federal portfolio will be the point where the dotted line and the solid effective cost line intersect.

Panel (a) plots the average cost of the portfolio absent all maturity and refinancing responses (using the maturity choices of borrowers charged risk-based interest rates). This is equivalent to a policy scenario in which the government forced individuals into a specific maturity, and shut down the refinancing channel. Because there are no maturity or refinancing responses, this cost stays constant for all values of $g$. If the government charged $g = 6.15\%$, the point where cost is equal to revenue, they would break-even on the portfolio.

Panel (b) plots the average cost of the portfolio allowing only for a maturity response, not a refinancing response. This is equivalent to our baseline scenario, in which all borrowers were pooled in the federal portfolio. Note that as the government charges a lower $g$, individuals extend their maturity which increases the average cost. As they increase $g$ individuals decrease their maturity, which decreases the average cost. The break-even point is slightly higher, $g = 6.39\%$, then in the case where there was no maturity response.

Panel (c) plots the average cost of the portfolio allowing only for a refinancing response, not a maturity response. The average cost curve is always upward sloping in $g$, since as $g$ increases the lowest risk individuals in the pool will refinance, increasing the average cost of the remaining pool. This means that it will be impossible to break-even on the portfolio once the refinancing channel is open, but it is possible to minimize the size of the subsidy provided to the remaining borrowers. At $g = 7\%$, the subsidy is minimized at 0.22%; moving from $g = 6.39\%$ to $g = 7\%$ reduces the interest rate subsidy by .12% from 0.34% to .22%.

Panel (d) plots the average cost of the portfolio allowing for both a refinancing response and a maturity response, the scenario closest to current policy. Individuals, especially higher risk individuals, choose a longer maturities under the flat, uniform $g$ price schedule.
This increases the effective subsidy size for smaller values of $g$, and suggests that the government could minimize the cost of this subsidy by increasing the uniform rate slightly above the rate that assumes away all behavioral responses, to 8.27%.

Figure 9: Federal Loan Portfolio Cost vs. Revenue by Uniform Interest Rate, Accounting for Behavioral Responses

These graphs analyze how revenue and costs vary for the government with the uniform interest rate that they charge, $g$, and with various behavioral responses by borrowers. The horizontal axis represents the different values of a uniform interest rate $g$, that the government could charge and collect on federal loans. On the vertical axis, I show how the amount that the government will raise and spend on the federal loan portfolio, accounting for heterogeneous risk types, maturity choices, and refinancing choices, varies with $g$. The dotted lines, on the 45 degree angle, plots the uniform interest rate being charged. This is effectively the revenue the government will collect for a given value of $g$. The solid lines plot the average cost the government faces for a given value of $g$. Therefore, the net interest rate tax or subsidy provided by the government at any given value of $g$ can be found by tracing out the vertical distance between the dotted line and the solid line at that point. The value of $g$ that will allow the government to break even on the federal portfolio will be the point where the dotted line and the solid effective cost line intersect.
6 Conclusion

Risk based pricing has advanced in many lending and insurance markets, with real implications for sorting and surplus. In car insurance, firms now use data from tracking devices to cherry-pick the lowest risk drivers. In the private student loan origination market in 2011, 40% of new borrowers had FICO scores greater than 770, while less than 5% had scores below 670. At times, the government acts as a concurrent source of credit or a regulatory body: in the mortgage market, FHA-backed loan eligibility is predicated on risk-related factors like credit score, with government-provided subsidies for lower income households. In the health insurance market, the government has limited the set of risk-related factors that can determine premiums. This paper provides a framework to analyze and inform the role of government in such settings.

I study how the evolution of risk-based pricing in the student loan market impacts borrower welfare and government revenue. I show that without a private refinancing option, the governments’ uniform interest rate policy achieves modest redistribution over income, but generates sizable distortions in borrowers’ intertemporal consumption decisions.\(^{29}\) By refinancing into a risk-based interest rate, the average low-risk borrower can increase surplus by $1530, but the government will need to subsidize the remaining high-risk borrower on average $1507 to maintain equity.

In the student loan space, a subsidy policy will stem unraveling and fulfill more precise redistributive motives – rather than implicitly “taxing” low risk borrowers under a break-even interest rate, the transfer could be funded by an income tax, by individuals who are not necessarily borrowers, and allow for intergenerational redistribution. In contrast, the findings suggest that a regulatory policy would primarily reduce efficiency, not inequity. I show that when firms price can only on FICO score, they reduce the gains to low risk borrowers substantially but do not reduce the extent of selection into the private market.

In addition to policy takeaways, this paper presents a novel micro-analysis of student borrowers. It shows there is demand for flexible repayment structures (like a maturity continuum) that allow households to distribute payments optimally over time, and con-

\(^{29}\)While this analysis focuses on cross-sectional redistribution under a uniform rate, the policy also redistributes consumption longitudinally— all borrowers were liquidity constrained when beginning school and unable to secure private credit at price below the Federal rate. Thus even ex-post low risk borrowers benefit from the uniform rate in the interim. The welfare implications are more complex in light of this dynamic selection problem.
siderable heterogeneity in borrowers’ desire to lower monthly payments vs. interest rates. While this analysis focuses on how interest rates impact repayment decisions, student borrowers could also respond to interest rate levels at earlier steps in the borrowing process, for example when taking out debt or deciding whether to attend graduate school. The availability of better risk-based interest rates could change these decisions, and is an interesting area for future work.
A Modeling Borrower Delinquency

A.1 Borrower problem: Impact of delinquency risk

In this section I solve the borrowers’ problem, allowing them to consider the impact of delinquency on their expected utility. Borrowers of risk type \( p \) and debt amount \( D \) choose maturity \( T \) to maximize:

\[
\max_T \sum_{i=1}^{Q} \mathbb{E}[u(c(t))]
\]

where \( Q \) is the individuals’ maximum age (i.e. finite). I define consumption as post-tax income \( w(t) \) minus a student debt payment \( d(T) \) if the loan is still being paid off.

The debt payment \( d(T) \), that individuals make each period is a function of their total debt amount \( D \), their chosen maturity, \( T \), and the risk, maturity-specific interest rate schedule that they are offered, \( r(T, p) \):

\[
d(T) = D \cdot \frac{r(T, p)}{(1 - (1 + r(T, p))^{-T})}
\]

In the borrowers’ maximization problem, agents can decrease their debt payment by increasing their maturity – i.e. \( \frac{dd(T)}{dT} < 0 \). This is the only “choice” variable that the borrower has – I assume that the interest rate schedule a borrower is offered is exogenous (determined by the lender) and therefore cannot be manipulated by the borrower to change their monthly payments or financing costs.

While borrowers can control their yearly payment level, they cannot control their variable, growing income stream which I parametrize as a unit root process (as in the main specification). Since income is variable, it is possible that a bad income shock could force consumption as currently defined \( (w(t) – d(T)) \) to a negative level. Therefore I specify a certain minimum threshold \( (x) \) that consumption never falls below. If ever \( (w(t) – d(T) < x) \), an individual will not pay their entire debt payment in that period and instead consume \( x \), creating a discontinuity in the consumption function. Specifically, the consumption function is given by:
\[ c(t) = (w(t) - d(T)) \ast \mathbb{1}(w(t) - d(T) \geq x) + x \ast \mathbb{1}(w(t) - d(T) < x) \quad \text{for } t \leq T \]
\[ c(t) = w(t) \quad \text{for } t > T \]

If a borrower ever misses a payment (or part of a payment) in a period, the borrower will have to continue making payments on the loan as scheduled for the remainder of the \( T \) periods and face a large utility penalty, which I denote as \( G \), after \( T \). If \( G \) is large enough, this will rule out the possibility of “strategic” default on the borrowers’ part.\(^{30}\) If an individual never misses a payment, they will consume their entire income in every period after period \( T \) and not face a penalty.

I assume that both the firm and the borrower have symmetric expectations about the borrowers’ income process, and therefore of \( \Pr(w(t) - d(T) < x) \). In the following section, I empirically back out the \( \Pr(w(t) - d(T) < x) \) that is implied from each borrower/maturity specific interest rate that is offered by the firm.

To simplify the maximization problem, I make a key assumption that the probability of delinquency does not change over time and is equal to the borrower’s initial delinquency risk; i.e. \( \Pr(w(t) - d(T) < x) = \Pr(w(0) - d(T) < x) = P(T) \).\(^{31}\) While \( P(T) \) does not change over time, I still allow the probability of delinquency to change with the chosen maturity; specifically, \( \frac{dP(T)}{dt} < 0 \), since a longer maturity will lower the the monthly payment in any given period.

I also make the delinquency penalty \( G \) linearly additive to utility from consumption.

\(^{30}\)Specifically, \( G \) must be large enough that totally utility after missing a payment is smaller than total expected utility without missing a payment.

\(^{31}\)Note that:
\[
\Pr(w(0) - d(T) < x) = \Pr(w(0) < x + d(T)) \\
= \Pr(ln(w(0)) + u_s < ln(x + d(T))) \\
= \Pr(u(t) < ln(x + d(T)) - ln(w(0))) \\
= \Phi(ln(x + d(T)) - ln(w(0))) \\
= \frac{\Phi(ln(x + d(T)) - ln(w(0)))}{\sigma_u(p)}
\]
These simplifications make the maximization problem:

$$\max_T \sum_1^T \mathbb{E}[u(w(t) - d(T))|w(t) - d(T) > x] \cdot (1 - P(T)) + u(x) \cdot P(T) + \sum_1^Q \mathbb{E}[u(w(t))] - G \cdot P(T) \cdot T$$

Differentiating the borrower problem with respect to $T$, we can see the exact impact of a change in maturity on expected utility through the first order condition. Allowing delinquency to enter the problem also adds two additional terms to the simplified first order condition:

$$0 = \sum_1^T \mathbb{E}[u(w(t) - d(T))|w(t) - d(T) > x] \cdot \frac{dd(T)}{dT} \cdot (1 - P) + \sum_1^T (u(x) - \mathbb{E}[u(w(t) - d(T))|w(t) - d(T) > x]) \cdot \frac{dP}{dT} + \mathbb{E}[u(w_T - d(T))|w_T - d(T) > x] \cdot (1 - P) + u(x) \cdot P - \mathbb{E}[u(w_T - d(T))]$$

An increase in maturity will now impact expected utility in four main ways:

- **It lowers the size of the monthly payment, which increases utility while paying off the loan (Effect A).** Because agents are hand-to-mouth, they can better smooth consumption with a lower monthly payment – there will be a small “jump” in utility when they finish paying off the loan. A second utility smoothing benefit comes from the fact that income is risky and a lower monthly payment provides some “insurance” value against more volatile income shocks. However, the interest rate $r(T)$ increases with maturity, so $d(T)$ is decreasing in $T$ at a *decreasing* rate.

- **It changes the length of repayment, which means the borrower has to pay the loan...**
off for one additional period, and this lowers total expected utility (Effect C).

- It decreases the probability of delinquency, $P(T)$, in any single period since there is a lower monthly payment. During repayment, this increases expected utility by lowering the chance that you consume at the threshold $x$ (Effect B). After repayment, this increases expected utility by lowering the chance that you have to pay the penalty $G$ (Effect D).

- It increases the probability of delinquency over the life of the loan since you are paying off over more periods. This increases the expected penalty $G$ (Effect D).

The main specification captures the consumption smoothing effects, $A$ and $C$, of a lower maturity; the delinquency threshold adds the additional effects, $B$ and $D$. The FOC allows one to say something about the size and sign of these effects: the size of effect $B$ is bounded mechanically – when $(w(t) - d(T))$ is small, $(E[u(w(t) - d(T))|w(t) - d(T) > x] - u(x))$ is close to zero. Conversely, when $(w(t) - d(T))$ is large, there is a small probability of delinquency and $\frac{dP}{dT}$ is near zero. This means effect $B$ is positive but small. Effect $D$ has an ambiguous sign and size. As maturity increases, each period the chance of delinquency becomes smaller; yet, the number of periods during which delinquency can occur increases. This means that $(P(T) + T \ast \frac{dP(T)}{dT})$ could be positive or negative. The size of $D$ also depends on the size of $G$, the delinquency “penalty”, which is unobserved.

In the empirical section I test how sensitive estimates of $\gamma$ are to the inclusion or exclusion of effect $D$. I calibrate $(P(T) + T \ast \frac{dP(T)}{dT})$ using the empirical delinquency probabilities implied by observed risk-based interest rates, and test the sensitivity of $\gamma$ to a wide range of values of $G$. The results suggests that the estimates are not very sensitive to the exclusion of the delinquency channel – for this low risk group, the first order effect of a change in maturity seems to operate instead through the monthly payment consumption smoothing channel.
B Derivation of Analytical First Order Condition:

Analytical Estimation:

We observe starting income levels $w_{i0}$. This means that we can express income at time $t$ as:

$$
\ln(w_{it}) = \ln(w_{i0}) + t \cdot (X'_i \mu) + \sum_{1}^{t} u_{it}
$$

$$
w_{it} = w_{i0} * e^{(X'_{i} \mu) + \sum_{1}^{t} u_{it}}
$$

If we return to the uncertain portions of the right hand side of our first order condition, $E[(w_{it} - d_i)^{-\gamma}]$, note that we can rewrite the expected marginal utility as the marginal utility of a certainty equivalent given by:

$$
E[(w_{i0} * e^{(X'_{i} \mu) + \sum_{1}^{t} u_{it}} - d_i)^{-\gamma}] = (w_{i0} * e^{(X'_{i} \mu) + \pi_{it} - d_i})^{-\gamma}
$$

where $\pi_{it}$ is the certain amount an individual would have to be given in that period to make their certain utility equivalent to the expected utility. Specifically:

$$
\pi_{it} = \frac{1}{2} * t * \sigma^2 [1 - (1 + \gamma) \frac{w_{i0} * e^{(X'_{i} \mu)}}{w_{i0} * e^{(X'_{i} \mu) - d_i}}] \quad \text{for } t < T+1
$$

$$
\pi_{it} = \frac{1}{2} * t * \sigma^2 (-\gamma) \quad \text{for } t \geq T+1
$$

To derive $\pi_{it}$, note that we can write:

$$
E[u'(w_{it})] = E[u'(w_{i0} * e^{(X'_{i} \mu) + \sum_{1}^{t} \sigma \epsilon_{it}})]
$$

where $\epsilon_{it} \sim N(0,1)$. We want to find the value of $\pi(\sigma)$ that allows us to write:

$$
E[u'(w_{i0} * e^{(X'_{i} \mu) + \sum_{1}^{t} \sigma \epsilon_{it}} - d_i)] = u'(w_{i0} * e^{(X'_{i} \mu) + \pi(\sigma) - d_i})
$$
For simplicity, start with the case of no income growth in period 1.

\[ E[u'(w_{i0} * e^{\sigma \epsilon_{1i}} - d_i)] = u'(w_{i0} * e^{\pi(\sigma)} - d_i) \]

We first take the derivative of this expression w.r.t. \( \sigma \):

\[ E[w_{i0} * e^{\sigma \epsilon_{1i}} u''(w_{i0} * e^{\sigma \epsilon_{1i}} - d_i)] = \pi''(\sigma) w_{i0} * e^{\pi(\sigma)} u''(w_{i0} * e^{\pi(\sigma)} - d_i) \]

At \( \sigma = 0 \) this becomes zero since \( E[\sigma \epsilon] = 0 \) and thus \( \pi'(0) = 0 \).

We next take the second derivative of this expression w.r.t. \( \sigma \), and evaluate it at \( \sigma = 0 \):

\[ E[\epsilon^2 u''(w_{i0} - d_i) + \epsilon^2 w_{i0} u''(w_{i0} - d_i)] = \pi''(0) u''(w_{i0} - d_i) \]

Under the assumption of CRRA utility, this becomes:

\[ \pi''(0) = [1 + w_{i0} \frac{u''(w_{i0} - d_i)}{w''(w_{i0} - d_i)}] \]

We now have a value for \( \pi''(0) \). This is helpful when evaluating a Taylor expansion of \( \pi(\sigma) \):

\[ \pi(\sigma) \approx \pi(0) + \pi'(0) \sigma + \frac{1}{2} \sigma^2 \pi''(0) \]

\[ \pi(\sigma) \approx \frac{1}{2} \sigma^2 [1 - (1 + \gamma) \frac{w_{i0}}{w_{i0} - d_i}] \]

Therefore our analytical estimating moment becomes:

\[ g_i(\theta) = \sum_{t=1}^{T} \beta^t \frac{\partial d}{\partial T} (w_{i0} * e^{t(X_i^\mu)} * e^\pi_{11} - d_i)^{-\gamma} - \beta^{T+1} (w_{i0} * e^{(T+1)(X_i^\mu)} * e^\pi_{T+1})^{-\gamma} \]

To estimate the model, I use nonlinear least squares, choosing the parameters that
minimize the quadratic form:

\[
b = \arg \min_{\theta} g_i(\theta)'g_i(\theta)
\]
C Evidence of Price Competition in the Private Sector

In the counterfactual, I make the assumption that the private refinancing market is perfectly competitive. Importantly, this assumption means that there will be no changes in producer surplus ($PS = 0$) during the counterfactual, only consumer surplus. On the surface, refinancing market displays most features of perfect competition, including rapid entry into the industry by many firms, and little product differentiation. It is also very easy for consumers to price shop and compare across refinancing firms, due to their online nature and the fact that they all offer quick, personalized price “quotes”.

In this section I provide empirical evidence of very strong price competition by estimating empirical refinancing elasticities with respect to quoted APR. I estimate the model using the logistic regression framework:

$$Pr(R_i = 1) = X_i'\mu - \beta r(10, p)_i + v * p_i + \epsilon_{ij}$$

The dependent variable ($R_i$) indicates whether an individual refinanced with this specific refinancer after seeing an interest rate quote. The dependent variable of interest is the 10 year fixed APR ($r(10, p)_i$) an individual was offered. I again isolate and use only the firm-conducted price changes as a source of $r(10, p)_i$ variation, controlling directly for risk type in the regression. The price elasticity is therefore identified only off of within risk type variation that happened at this specific refinancing firm.

If price competition is high, then a small increase in $r(10, p)_i$ at this single firm should create a large decrease in $Pr(R_i = 1)$ at this firm. Using $\beta$ to measure price competition assumes that the within-risk type price shifts were made independently of price changes at other firms and the Direct Loan program. This assumption seems reasonable, since the price changes were conducted to collect information on borrower elasticities, and not for competitive reasons. If they were made in tandem with other firms, this will bias our elasticity downwards.

The results of the main specification are shown in column 1 of Table 6, expressed as both elasticities and marginal effects. They provide evidence of strong price competition: when the interest rate at this one firm increases by 1%, borrowers are 5.6% less likely to refinance at this specific firm.

Column 1 uses only within risk-type price variation, where as the regression in column
### Table 6: Extensive Margin Refinancing Elasticities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Within Risk Type Variation Only*</th>
<th>Within and Over Risk Type Variation**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>-5.63</td>
<td>-5.33</td>
</tr>
<tr>
<td>( dy/dx \times (x/y) )</td>
<td>(.182)</td>
<td>(.070)</td>
</tr>
<tr>
<td>Marginal Effect</td>
<td>-.00103</td>
<td>-.00098</td>
</tr>
<tr>
<td>( dy/dx )</td>
<td>(.000033)</td>
<td>(.00001)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \text{Prob. Refinance} )</td>
<td>.05</td>
<td></td>
</tr>
<tr>
<td>( x = 10 \text{ Yr Fixed APR (bps)} )</td>
<td>562.56</td>
<td>57.42</td>
</tr>
</tbody>
</table>

N \(= 202930 \)  \(= 202930 \)

Standard errors in parentheses. Specification are logistic regressions of dummy variable if individual fully refinances their loan at this company on the 10 year fixed APR they were offered. Results are expressed both as marginal effects, the unit change in \( y \) for a unit change in \( x \), and as elasticities, the proportional change in \( y \) for a proportional change in \( x \). Means and std. of the dependent and independent variable are shown in the bottom rows.

*Regression controls for risk type at the level of within risk type price variation (categorically). Therefore, variation in the 10 yr fixed APR comes only from over-time shifts in the price schedule.

**Regression does not control for risk type. Therefore, variation in the 10 yr fixed APR comes from both over-time shifts in the price schedule and risk-based prices.

2 does not control for risk type and therefore uses both within and across risk type price variation. It is interesting to note that both find a very similar elasticity, which suggests that the extensive margin sensitivity to refinance does not differ over risk type. This helps support our assumption during the maturity analysis that the results were not driven by compositional changes; i.e. changes in the sample were not correlated with taste for maturity.
## Additional Figures and Tables

### Table 7: Budget Lifetime Default Rates

<table>
<thead>
<tr>
<th>Year Loan Enters Repayment</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Year Non-Profit/Public</td>
<td>32.10%</td>
<td>31.60%</td>
<td>31.10%</td>
<td>31.40%</td>
<td>33.80%</td>
</tr>
<tr>
<td>2-Year Proprietary</td>
<td>47.30%</td>
<td>48.60%</td>
<td>49.00%</td>
<td>48.40%</td>
<td>49.40%</td>
</tr>
<tr>
<td>4-Year Freshmen &amp; Sophomores</td>
<td>24.70%</td>
<td>24.00%</td>
<td>23.60%</td>
<td>24.20%</td>
<td>25.40%</td>
</tr>
<tr>
<td>4-Year Juniors &amp; Seniors</td>
<td>12.40%</td>
<td>12.30%</td>
<td>12.10%</td>
<td>11.90%</td>
<td>13.00%</td>
</tr>
<tr>
<td>Graduate Students</td>
<td>6.20%</td>
<td>6.20%</td>
<td>6.10%</td>
<td>6.10%</td>
<td>6.40%</td>
</tr>
<tr>
<td><strong>Weighted Average</strong></td>
<td>15.90%</td>
<td>16.50%</td>
<td>17.30%</td>
<td>17.60%</td>
<td>18.40%</td>
</tr>
</tbody>
</table>

Source: U.S. Department of Education (based on figures published in fiscal year 2014)
Table 8: Maturity Choice Regression

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed 60</td>
<td>-4.506</td>
</tr>
<tr>
<td></td>
<td>(3.313)</td>
</tr>
<tr>
<td>Fixed 120</td>
<td>-15.32***</td>
</tr>
<tr>
<td></td>
<td>(5.149)</td>
</tr>
<tr>
<td>Variable 120</td>
<td>10.18***</td>
</tr>
<tr>
<td></td>
<td>(2.141)</td>
</tr>
<tr>
<td>FICO</td>
<td>-0.0706***</td>
</tr>
<tr>
<td></td>
<td>(0.0156)</td>
</tr>
<tr>
<td>1 Risk Score</td>
<td>-21.58**</td>
</tr>
<tr>
<td></td>
<td>(10.16)</td>
</tr>
<tr>
<td>2 Risk Score</td>
<td>-26.16**</td>
</tr>
<tr>
<td></td>
<td>(10.53)</td>
</tr>
<tr>
<td>3 Risk Score</td>
<td>-35.07***</td>
</tr>
<tr>
<td></td>
<td>(11.16)</td>
</tr>
<tr>
<td>4 Risk Score</td>
<td>-40.13***</td>
</tr>
<tr>
<td></td>
<td>(11.78)</td>
</tr>
<tr>
<td>5 Risk Score</td>
<td>-45.29***</td>
</tr>
<tr>
<td></td>
<td>(12.45)</td>
</tr>
<tr>
<td>6 Risk Score</td>
<td>-44.88***</td>
</tr>
<tr>
<td></td>
<td>(13.20)</td>
</tr>
<tr>
<td>7 Risk Score</td>
<td>-45.35***</td>
</tr>
<tr>
<td></td>
<td>(13.73)</td>
</tr>
<tr>
<td>8 Risk Score</td>
<td>-48.31***</td>
</tr>
<tr>
<td></td>
<td>(14.13)</td>
</tr>
<tr>
<td>9 Risk Score</td>
<td>-52.13***</td>
</tr>
<tr>
<td></td>
<td>(14.79)</td>
</tr>
<tr>
<td>10 Risk Score</td>
<td>-59.02***</td>
</tr>
<tr>
<td></td>
<td>(15.51)</td>
</tr>
<tr>
<td>11 Risk Score</td>
<td>-55.87***</td>
</tr>
<tr>
<td></td>
<td>(16.04)</td>
</tr>
<tr>
<td>12 Risk Score</td>
<td>-59.70***</td>
</tr>
<tr>
<td></td>
<td>(16.59)</td>
</tr>
<tr>
<td>ln(amt)</td>
<td>25.46***</td>
</tr>
<tr>
<td></td>
<td>(0.638)</td>
</tr>
<tr>
<td>Monthly FCF</td>
<td>-0.00263***</td>
</tr>
<tr>
<td></td>
<td>(0.000284)</td>
</tr>
<tr>
<td>BA/BS</td>
<td>-1.143</td>
</tr>
<tr>
<td></td>
<td>(4.032)</td>
</tr>
<tr>
<td>Masters</td>
<td>-12.80***</td>
</tr>
<tr>
<td></td>
<td>(4.072)</td>
</tr>
<tr>
<td>Professional</td>
<td>-13.28***</td>
</tr>
<tr>
<td></td>
<td>(4.088)</td>
</tr>
<tr>
<td>Number Mortgages</td>
<td>2.390***</td>
</tr>
<tr>
<td></td>
<td>(0.839)</td>
</tr>
<tr>
<td>1 Dependents</td>
<td>5.848***</td>
</tr>
<tr>
<td></td>
<td>(1.325)</td>
</tr>
<tr>
<td>2 Dependents</td>
<td>10.57***</td>
</tr>
<tr>
<td></td>
<td>(1.629)</td>
</tr>
<tr>
<td>3 Dependents</td>
<td>10.70***</td>
</tr>
<tr>
<td></td>
<td>(2.597)</td>
</tr>
<tr>
<td>4 Dependents</td>
<td>15.11***</td>
</tr>
<tr>
<td></td>
<td>(4.351)</td>
</tr>
<tr>
<td>5 Dependents</td>
<td>11.21</td>
</tr>
<tr>
<td></td>
<td>(7.717)</td>
</tr>
<tr>
<td>Constant</td>
<td>19.99</td>
</tr>
<tr>
<td></td>
<td>(43.61)</td>
</tr>
</tbody>
</table>

| N                | 10680             |

Standard errors in parentheses
Regression also controls for dummyed age, and binned income. Omitted degree is BA. The firm-specific risk score was split into 13 bins, and these dummies are represented by the categories "1 Risk Score" etc, where 1 is the highest risk category and 12 is the lowest.

* p < 0.10, ** p < 0.05, *** p < 0.01
Table 9: Test of Extensive Margin Response and Changes in Borrower Composition

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>SE</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>APR *</td>
<td>-3.30</td>
<td>{ -8.07 , 1.466}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>10680</th>
</tr>
</thead>
</table>

Response of $\hat{T}$ to APR
<table>
<thead>
<tr>
<th>Observed Variables</th>
<th>Definition</th>
<th>Data Used</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_0 )</td>
<td>Annual Income</td>
<td>Observed Post-tax Annual Income</td>
<td>81,860</td>
<td>70,356</td>
<td>44,556</td>
</tr>
<tr>
<td>( d(T) )</td>
<td>Yearly Debt Payment at Maturity ( T )</td>
<td>Yearly Student Loan Payment at Maturity ( T )</td>
<td>11,005</td>
<td>7,980</td>
<td>9,006</td>
</tr>
<tr>
<td>( T )</td>
<td>Loan Maturity</td>
<td>Maturity in Months</td>
<td>85</td>
<td>60</td>
<td>44</td>
</tr>
<tr>
<td>( r(T) )</td>
<td>Risk-based Interest rate at Maturity ( T )</td>
<td>Observed ( r(T) ) %</td>
<td>4.95</td>
<td>5.04</td>
<td>0.90</td>
</tr>
<tr>
<td>( i(T) )</td>
<td>Risk-Free Interest Rate at Maturity ( T )</td>
<td>Lowest Risk Type ( r(T) - \epsilon ) in %</td>
<td>3.88</td>
<td>3.50</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Definition</th>
<th>Calibration Value Used</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Recovery Rate on Per-Period Payment</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>Min. Consumption Threshold</td>
<td>Yearly Housing Expense</td>
<td>18,414</td>
<td>16,032</td>
<td>13,736</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Definition</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(T) )</td>
<td>Per-Period Delinquency Probability</td>
<td>0.046</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>Standard Deviation of ( ln(w) )</td>
<td>0.57</td>
<td>0.29</td>
<td>1.12</td>
</tr>
</tbody>
</table>

*Table 10: Empirical Income Volatility Exercise: Calibration Values and Estimates*
### Figure 10: Description of Federal Loan Repayment Plans

<table>
<thead>
<tr>
<th>Eligible Loan Types</th>
<th>Term</th>
<th>Monthly Payment</th>
</tr>
</thead>
</table>
| **Standard Fixed Repayment** | • Direct Subsidized and Unsubsidized Loans  
• Subsidized and Unsubsidized Federal Stafford Loans  
• All PLUS loans | • Up to 10 Years | • Fixed amount, at least $50 per month |
| **Graduated Repayment** | • Direct Subsidized and Unsubsidized Loans  
• Subsidized and Unsubsidized Federal Stafford Loans  
• All PLUS loans | • Up to 10 Years | • Payments are lower at first and then increase every two years |
| **Extended Repayment** | • Direct Subsidized and Unsubsidized Loans  
• Subsidized and Unsubsidized Federal Stafford Loans  
• All PLUS loans | • Up to 25 Years | • Payments may be fixed or graduated |
| **Income Based Repayment** | • Direct Subsidized and Unsubsidized Loans  
• Subsidized and Unsubsidized Federal Stafford Loans  
• All PLUS loans made to students  
• Consolidation Loans (Direct or FFEL) that do not include Direct or FFEL PLUS loans made to parents  
• Discretionary income | • Up to 25 Years  
• If loan not repaid in full after making the equivalent of 25 years of qualifying monthly payments, any outstanding balance will be forgiven. | • Maximum payment is 15% of discretionary income  
• Discretionary income is the difference between your AGI and 150% of the poverty guideline  
• Payments change as income changes  
• Individual needs to demonstrate partial financial hardship to qualify |
| **Pay as You Earn Repayment** | • Direct Subsidized and Unsubsidized Loans  
• Direct PLUS loans made to students  
• Direct Consolidation Loans that do not include (Direct or FFEL) PLUS loans made to parents  
• Must be a new borrower on or after Oct. 1, 2007, and must have received a disbursement of a Direct Loan on or after Oct. 1, 2011. | • Up to 20 Years  
• If loan not repaid in full after making the equivalent of 20 years of qualifying monthly payments, any outstanding balance will be forgiven. | • Maximum payment is 10% of discretionary income  
• Payments change as income changes  
• Individual needs to demonstrate partial financial hardship to qualify |

Source: [https://studentaid.ed.gov/repay-loans/understand/plans](https://studentaid.ed.gov/repay-loans/understand/plans)

This table describes the various repayment plans available for Federal Direct Loans as of 2015.
Figure 11: Comparison of Applicant Pool to Nationally Representative Sample of Graduate Student Borrowers

This figure compares the student loan amount and income quantiles of my applicant pool to those in a nationally representative sample of graduate student borrowers. The two populations look very similar, which suggests that the individuals I observe are similar to graduate students with Federal Loans.

Figure 12: Using Across vs Within Risk Price Variation to Identify Term Elasticities

This figure shows the importance of only using within risk price variation to identify price elasticities. Panel (a) shows the term choices of two different risk types facing risk-based price variation – despite facing higher interest rates, the riskier type chooses a longer loan due to other omitted factors. This makes it seem as though term demand is increasing in interest rates. However, panel (b) shows that when prices increase within risk type, term demand actually decreases with interest rate.
Figure 13: Distribution of calculated values of $\hat{P}(T)$

Figure 14: Implied post-tax income ranges, using median estimated income variance $\sigma_u$. For each post-tax yearly income level (x-axis), this graph plots the range +/- 1 standard deviation and +/- 2 standard deviations around that income level, using the median income variance $\sigma_u^2$ estimated from the lender’s problem. Given the log-normal income assumption, there is a 68 percent chance that income each year will fall within the 1 S.D. range, and 95 % chance that income each year will fall within the 2 S.D. range.
The CDR is calculated and published by the Federal government at the school level, and reflects the student loan default rate of a cohort of students from that school after 3 years of completion. It is a much cited measure of expected costs used by Federal loan program. This figure compares the difference in the CDR between the highest and lowest risk types in my sample (which is roughly 2 percentage points) to the spread in their risk-based interest rates, and shows that private sector risk scores are highly correlated with the CDR.

This graph shows changes in three important observable characteristics, income, debt amount, and FICO score, over 10 price regimes. While there are differences across price regimes, it is comforting to note that there are no obvious monotonic trends in these three variables and that they are not correlated with the exogenous price shifts.
Figure 17: Simulated Relationship between Optimal Term Choice and offered Interest Rate
This figure shows how the simulated optimal term choice varies with interest rate levels for two values of the IES – as the level of interest rates increases, the optimal maturity choice decreases. For any given interest rate, individuals with a higher intertemporal elasticity of substitution will choose shorter optimal maturity. The graph shows, for example, the optimal maturity choices of two otherwise identical individuals when facing a uniform interest rate of 6% are almost 70 months apart. The simulation is for individuals with a loan amount of $90,000 and annual income of $50,000.

Figure 18: Average Size of Extra Payments Over Time Made by Borrowers
I look at payment patterns over time within my sample of refinancers – in other words, do any individuals change their payment level over time permanently, or do they systematically make higher or lower payments on their debt. I find that there are some extra payments in the data, but they are small and do not vary systematically over time. This supports our model’s assumption that borrowers make a term choice in year 1 to maximize expected utility over the life of the loan and are not in fact choosing a monthly payment to fit their current income level, with the intent to refinance and change term yet again in the future when their income level changes.
Figure 19: Estimated log income growth rates

Figure 20: Levels and Changes in Other Monthly Payments Before and After Refinancing

My model assumes that borrowers are not readjusting on other financial margins when refinancing. In other words, contemporaneous savings and debt decisions are assumed to be exogenous, predetermined, and unaffected by maturity and refinancing decisions. Here I test this assumption by looking at borrowers’ other monthly payments before and after refinancing, and measuring whether they adjust immediately during refinancing. This table describes changes in other monthly payments (mortgages, auto loans, credit cards, etc) before vs. after refinancing for individuals who had positive monthly payments to begin with, and shows that for the vast majority of borrowers these stayed constant. This makes sense, since many of these payments are fixed installments, and it would take active work on the borrower’s part to readjust.
Figure 21: Investment Balances over Lifetime

My model defines yearly consumption as income minus the student debt payment; in reality individuals may also be making savings decisions that could impact their maturity choices. I can observe the savings and investment behavior of borrowers in my sample: because individuals in my sample are young, they have relatively low levels of savings to begin with. Slightly under 40% have a formal retirement savings account – for example 25% have a 401k, with a median balance of $24,000. The number of individuals with investment holdings increases with age. This figure shows that while the median borrower continues to not have substantial savings through age 60, the 75th percentile has accumulated over $80,000 by age 50. However, 90% of my borrowers are under 40 years old, and therefore even the most active savers have investment holdings that are much smaller than their student debt amount.