The Impact of Risk-Based Pricing in the Student Loan Market: Evidence from Borrower Repayment Decisions

Natalie Bachas*

December 19, 2019

Abstract

Advances in credit underwriting have both efficiency and equity implications. In the $1.4 trillion student loan market, private lenders offer borrowers risk-based interest rates, while the federal loan program sets a uniform price. I measure changes in consumer surplus that occur as low-risk types refinance out of the government pool into the private market. I use a dataset from an online refinancer to estimate a structural model that relates borrowers’ monthly payment choices to interest rates. I estimate refinancing increases low-risk surplus by $1,302, and show substantial distortionary costs (32% of the average transfer) under a pooled, uniform interest rate.

JEL Classification: G21, G51, I22

---

*Princeton University, Bendheim Center for Finance, 20 Washington Rd, Princeton, NJ 08540, E-mail: nbachas@princeton.edu. I thank my committee members, Ben Handel, David Sraer, Ulrike Malmendier and Emmanuel Saez, who provided invaluable advice throughout the research process. This work has benefited particularly from the comments of Alan Auerbach, Pierre Bachas, Zarek Brot-Goldberg, Patrick Kline, Jon Kolstad, Julien Lafortune, Waldo Ojeda, Avner Schlain, Wei Xiong, Danny Yagan, Motohiro Yogo, Gabriel Zucman and seminar participants at Princeton University, Stanford GSB, Yale, Columbia GSB, MIT Sloan, Harvard, HBS, BYU, Booth, Wharton. I also thank the patient and helpful employees at the firm whose data I analyze.
1 Introduction

Risk-based pricing has increased in use and sophistication, especially as more lenders and insurers transition to an online setting. It has been shown in theory and practice that the ability to identify and accurately price consumer risk can generate large efficiency gains in markets with selection.\(^1\) However, if advances in risk-based pricing create clear winners and losers, this will also change the government’s role in ensuring equity and redistribution of surplus.

This is especially true in the student loan refinancing market, where private firms use rich financial and educational data to underwrite student borrowers who have finished school. These individualized prices contrast with those of the Federal Direct Loan program, which offers borrowers a uniform interest rate despite observable variation in the expected costs of lending. Moving from average to marginal cost pricing could correct allocative inefficiencies - low risk types might increase borrowing when faced with an undistorted price in the private sector. But pricing innovations could also have complex implications for how private and public lending options coexist. As low risk types refinance into the private sector, the average risk of the remaining federal borrowers will rise, forcing the government to either raise its uniform rate or subsidize the remaining pool.\(^2\)

This paper studies this efficiency-equity tradeoff empirically, using a dataset of applicants from an online student loan refinancing firm that employs comprehensive risk-based pricing. Using a series of firm-conducted price changes, I show that observa-

---

\(^1\) Several papers (Einav et al., 2012, 2013; Edelberg, 2006; Finkelstein and Poterba, 2014) show that credit-scoring can generate both efficiencies in consumer lending, and impact market structure. There is also a literature that studies uniform and average cost pricing schemes in the presence of heterogeneous risk (Bundorf et al., 2012; Nelson, 2018; Hurst, 2016) in several markets (health insurance, mortgages) that has shown that while uniform pricing policies achieve cross-sectional redistribution, they can also distort consumer choices and generate welfare loss.

\(^2\) While prepayment penalties are outlawed in the student loan market, (Mayer et al., 2013) studies a similar dynamic in the mortgage market, and argues that prepayment penalties can be welfare improving if they prevent ex-post low risk types from leaving the pool once risk has been realized.
tionally similar borrowers are interest rate sensitive during repayment: they increase monthly payments and shorten maturity when interest rates increase. This suggests that there will be a distortionary “cost” to the governments’ one-size-fits-all pricing policy as it transfers from low to high risk borrowers. I estimate and combine this monthly payment elasticity with the observed distribution of risk based prices to quantify the deadweight loss and redistribution that occurs under a uniform interest rate, as well as the budgetary impact of low risk borrowers refinancing into the private sector.

My empirical analysis uses a unique dataset that contains maturity choices, interest rate variation, and information on borrowers’ income and balance sheets. Each borrower in the sample is faced with a menu of repayment maturities and corresponding, increasing risk-based interest rates. They choose a specific maturity on a continuum of 60 to 240 months.

This dataset allows for two novel contributions: first I describe how borrower characteristics influence monthly payment choices. While existing data sources, such as credit bureau data, contain information on monthly payments, they are unable to map those choices to heterogeneity in borrowers’ income or cashflow. I find that the monthly debt to income ratio is the central determinant of maturity choice – borrowers in my sample consistently aim for a payment that is 30-40% of their monthly free cash flow, which is defined as income net of taxes and fixed expenditures such as rent. Conditional on income, maturity choice increases with age, number of dependents, total liabilities, and having a mortgage. It decreases with degree of education and FICO score, but not with total assets.

I then study the inherent tradeoff borrowers face between the level of monthly payment and the overall amount of interest incurred on their debt. When interest rates increase, borrowers may reduce maturity via the substitution effect to effectively "borrow less”. An inelastic response suggests that borrowers value a longer maturity and low immediate monthly payment, even if it requires paying a higher price. I use ex-
ogenous variation in offered interest rate schedules to estimate the elasticity of maturity choices to interest rates. While I cannot observe the same borrower making choices under two different price sets, the repeated cross-sectional nature of the data means that I can compare the choices of observably similar borrowers (matched on characteristics like debt, income, risk, and age) who are offered different prices. Reduced form evidence that regresses interest rates on maturity choice reveals that the borrowers in my sample are interest rate sensitive, decreasing maturity when interest rates increase, and that this sensitivity increases with borrower income and liquidity. I find that borrowers who are more liquidity constrained, as proxied by monthly income, have a more inelastic demand for maturity: they tolerate a larger increase in interest costs in order to keep maturity and monthly payment relatively constant. In contrast, wealthier borrowers have a preference for very short maturities and higher price sensitivity.

Under a certain set of assumptions, the reduced form maturity elasticity maps to a familiar intertemporal consumption tradeoff. I develop and fit a stylized model in which hand-to-mouth consumers choose a repayment maturity to maximize expected utility. In the model borrowers vary in their level and volatility of income and thus in the consumption smoothing benefits of a long maturity loan. This heterogeneity generates differences in willingness to pay for maturity across risk types, as well as differences in lending costs. Matching the model’s first order condition to observed maturity choices, I estimate a disperse distribution of preferences and find a moderately high average IES of .55.

Using the estimated repayment model, I compare the size and distribution of borrower surplus under several pricing regimes: full pooling under a break-even uniform rate, pooling with a refinancing option using FICO-based pricing, and pooling with a comprehensive risk-based refinancing option. The first average-cost interest rate policy redistributes roughly $1,200 from low to high risk borrowers, but generates an average distortionary cost of $450, or 32% of the average transfer. The policy achieves more
modest redistribution over income quantiles given that borrower risk type is an imperfect proxy for borrower income. When low-risk types have the option to refinance into the private sector they gain on average $1,500 in surplus – they face a lower absolute level of interest rates, and respond by making smaller monthly payments. Both the distribution of risk based interest rates and the gains to low risk borrowers are much larger when firms price on borrower characteristics, like savings, income, and education, in addition to FICO score. This suggests that non-traditional scoring methods can benefit individuals who are low risk, but have underdeveloped credit histories (i.e. the student borrower population).

These findings highlight how developments in the private sector’s ability to price borrower risk will simultaneously i) improve welfare for low risk borrowers and ii) increase sorting of low risk borrowers out of the public repayment pool. I analyze one government policy response that would prevent unraveling and maintain equity in the federal pool: providing a net interest rate subsidy. My model highlights how the effective costs of an interest rate subsidy can deviate from the mechanical costs once refinancing and maturity responses are accounted for. For example, lowering the uniform interest rate will reduce refinancing into the private sector, improve the average risk of the remaining federal borrowers, and therefore decrease the subsidy’s average cost.

Both the descriptive and causal findings also suggest that there are two divergent groups of borrowers in repayment: those with high debt to income ratios who have a monthly payment focus, and those with excess liquidity that have an interest rate focus. Both the optimal repayment behavior (extended vs prepayment) and policy prescriptions for either group are also distinct. The elasticity results suggest that many borrowers have a high willingness to pay, and therefore value, for extended repayment programs, and that potentially some of the costs of these programs could be offset by a higher interest rate. Meanwhile, programs that allow borrowers to refinance their federal loans into

\[ \text{Currently federal extended and income-driven repayment programs do not have an interest rate that increases with maturity.} \]
shorter maturities or lower risk based interest rates would be greatly valued by the most elastic group, and may prevent the "leakage" of low risk borrowers out of the federal portfolio into the private refinancing sector.

This paper relates to several literatures: the question of how borrowing and consumption decisions respond to interest rates and are mediated by credit constraints is central to the household finance literature (Ganong and Noel, 2018; DeFusco and Paciorek, 2017; Gross and Souleles, 2002; Karlan and Zinman, 2009, 2018). I extend this discussion to the growing student debt market with a particular emphasis on how monthly payment decisions are influenced by liquidity and interest rates. Other papers that look specifically at maturity and/or monthly payment choices (Hertzberg et al., 2018; Argyle et al., 2019; Attanasio et al., 2008; Keys and Wang, 2019) also find that long maturities are preferred by riskier, liquidity-constrained borrowers. By using the maturity decision to estimate the IES, my paper contributes to a literature that structurally estimates parameters relating to risk aversion and consumption smoothing using micro-data on consumer choices and quasi-experimental variation in prices (Cohen and Einav, 2007; Einav et al., 2010; Best et al., 2015; Gruber, 2013). Finally, it furthers the study of student borrowers in repayment (Lochner and Monge-Naranjo, 2016; Lochner et al., 2018; Mueller and Yannelis, 2019; Herbst, 2019; Maggio et al., 2019), with a focus on how modified repayment terms impact both borrower welfare and the government budget.

2 Background and Setting

Interest Rates and Repayment Options: While this paper focuses on the repayment of student loans, it is first necessary to understand their origin. Aid that comes from the federal government to finance post-secondary education is by far the most popular option – over 90% of the student loan market consists of Federal Direct Loans.

Repayment of federal debt, and importantly choice of repayment plan, does not occur
until after a student has finished schooling. Borrowers also have the option of changing repayment plans as time progresses. Federal repayment plans fall into two general categories: fixed payment plans, which adjust the monthly payment level to ensure that the full amount of the original loan will be paid off in a specified number of years, and income-based plans, which scale the monthly payment in proportion to the borrower’s income. While borrowers are initially defaulted into a 10 year fixed payment plan, many borrowers opt into either income-driven or extended fixed repayment plans. Both effectively extend the maturity of the loan while lowering the monthly payment.

In contrast to the many borrowers who choose to extend repayment and lower monthly payments, a sizable fraction prepay their loans in order to generate interest rate savings.\textsuperscript{4} This is demonstrated in the high prepayment rates on student debt securities, and credit bureau evidence that shows almost 25\% of student loans are paid off within 1 year of starting repayment (Gibbs, 2017).

Federal interest rates follow a “one-size-fits-all” formula that is specified under the Higher Education Act. Specifically, each year they are determined by an index rate (currently the 10 year Treasury note), plus an add-on margin that varies by loan type (see Table 1). A report by the GAO notes that while these margins attempt to “break-even” in expectation, they have recently generated a loss for the Direct Loan program. Importantly, these rates do not vary with the risk of the individual borrower or the maturity of the loan.

A growing number of borrowers are now refinancing their federal loans in the private sector.\textsuperscript{5} Refinancing can take place at any point over the life of a loan - immediately

\textsuperscript{4}The absence of a prepayment penalty, means that even if a borrower is in a fixed payment plan the effective term of their loan might be much shorter.

\textsuperscript{5}The private refinancing sector has experienced rapid growth over the last 4 years – the first refinancing firms emerged in 2011, and since then the market has expanded to include more than a dozen major players. The majority of these firms are online lenders, who access applicants’ financial accounts and credit reports digitally, and use this extensive data to quickly and thoroughly underwrite risks. A report by Goldman Sachs in 2015 estimated that ”out of the $586 billion of Federal loans that are currently in repayment status... 25\% are eligible for refinancing (after eliminating lower credit quality customers that could not be offered a cheaper rate from a market-based lender), resulting in $147 billion of addressable
Table 1: 2011-2015 Interest Rates on Federal Direct Student Loans

This table lists government-specified interest rates on the three main types of federal student loans between 2008 and 2017. Interest rates vary by student type (i.e., graduate student vs. undergraduate), and are defined as an add-on plus the index rate (the 10-year treasury note). Undergraduates are able to borrow at lower interest rates up to a certain limit ($5,500 to $7,500, depending on their year in school), and then must borrow at higher interest rates. Source: http://www.ifap.ed.gov/eannouncements/051515DLInterestRates1516.html.

<table>
<thead>
<tr>
<th>Loan Type</th>
<th>Borrower Type</th>
<th>Index</th>
<th>Add-on</th>
<th>Fixed Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Un/Subsidized Loans</td>
<td>Undergraduate</td>
<td>10 Yr Tr</td>
<td>+ 2.05%</td>
<td>3.4-4.66%</td>
</tr>
<tr>
<td>Direct Unsubsidized Loans</td>
<td>Graduate/Professional</td>
<td>10 Yr Tr</td>
<td>+ 3.60%</td>
<td>5.41-6.8%</td>
</tr>
<tr>
<td>Direct PLUS Loans</td>
<td>Parents&amp; Graduate</td>
<td>10 Yr Tr</td>
<td>+ 4.60%</td>
<td>6.4-7.9%</td>
</tr>
</tbody>
</table>

when a borrower begins repayment to the Federal government, or in the midst of a repayment schedule. Federal loans, which do not carry a pre-payment penalty, are paid off by the private firm which takes over the servicing and liabilities associated with the loan. It is important to note that student loans that are refinanced in the private market are still not dischargeable in the case of bankruptcy. In contrast with the federal loan program, private lenders price loans with risk-based interest rates. These interest rates increase with the perceived risk of the individual and also with the maturity of the loan. This makes refinancing an attractive option for individuals who have good credit and/or want a shorter maturity.

The Maturity/Interest Rate Trade-off: Loan maturity, rather than loan size, is the relevant liquidity "lever" for student borrowers in repayment, since they have already locked in their principal decisions.\(^6\) Maturity choice is similar to a borrowing response: as interest rates increase (the cost of borrowing increases), borrowers potentially reduce maturity (i.e. borrow less) via a substitution effect.

When choosing a loan maturity, either in the private or public sector, borrowers trade lower monthly payments and immediate liquidity off against higher interest rates and total interest payments. Many borrowers are focused either on the monthly burden of a

Federal student loan market.” They also estimated another $64 billion of eligible private student loans existed, making for a total potential market of $211 billion.

\(^6\)In fact, the maturity decision is relevant for any consumer lending scenario in which principal is pre-determined or difficult to adjust.
loan, aka the payment made each month towards principal and interest, or on the total interest cost paid over the lifetime of that loan. When comparing loans of varying terms, these two entities (total cost and monthly payment) are inversely related – a longer term loan will have a lower monthly payment, but a higher overall interest cost (as more interest accumulates, at potentially a higher rate). This means that different loan terms will often appeal to different types of borrowers; for example, individuals with lower incomes who are more liquidity constrained may prefer a low monthly payment, despite additional interest costs.

Figure 1 illustrates this tradeoff for the average loan in my dataset - as the repayment contract length increases, yearly payments decrease while total interest paid increases. At the optimal maturity, borrowers balance the gain in marginal utility from having a slightly lower monthly payment over the life of the loan against the cost of paying more interest overall. Many factors impact this condition, like income level, debt amount, and the level of interest rates.

3 Data and Descriptive Evidence

To analyze consumer preferences for maturity, I use a dataset from a student loan refinancing firm that contains extensive information on prices, borrowers, loan balances, and maturity choices. The main sample that I use when measuring maturity elasticities is a repeated cross-section of all new borrowers refinancing with the firm over the period of a year; it links borrowers’ financial information (debt amount, income, assets, credit score) to the menu of interest rates they faced and the ultimate maturity choice they made when refinancing.

Borrowers repaying loans have traditionally been restricted to either a single 10 yr

\textsuperscript{7}The federal government charges a single interest rate for all loan terms, whereas the private sector charges an increasing rate for longer terms - this means that the "total" cost differential of a long vs. short term loan will be larger in the private refinancing sector than in the federal sector.
Figure 1: Tradeoff Between Monthly Payment, Maturity, and Interest Rate

Notes: This figure shows a stylized example of the tradeoff borrowers face when choosing a specific maturity. It is calibrated for a $60,000 loan, and uses the range of maturities and interest rates that are found in this setting. As the repayment maturity length increases, monthly payments decrease which increases cash on hand. However, the interest rate increases simultaneously, making borrowing more costly.

![Figure 1: Tradeoff Between Monthly Payment, Maturity, and Interest Rate](image)

term or a discrete set of term choices (10 yrs, 15 yrs, 20 yrs, etc). In my setting they are instead asked to choose from a continuum of terms from 5 to 20 years, allowing them customize their payments and providing the researcher with a more precise revelation of their repayment preferences.

Given the novelty of the choice set and complexity of interest rate/monthly payment tradeoff, one might wonder if borrowers are making completely informed decisions – for instance, if they only see how term impacts monthly payment and are unaware of the impact on interest rate, they may choose much longer term then they would have in a full information scenario. I argue there are several aspects of the user interface that borrowers interact with that make this unlikely: for one, borrowers are provided with calculations of monthly payment, APR, and total paid for the term the choose at many points during the refinancing process. Furthermore, borrowers use a "slider" to adjust their term, and are shown how monthly payments, APR, and total payments change simultaneously with term. This means they are aware not only of the tradeoffs inherent
when choosing any given term, but also the rate at which these tradeoffs change when they adjust term. Borrowers are also asked to contemplate and modify their term choice at several points during the refinancing process, which means their choices are unlikely to be remiss.

**Borrower Population:** This population of borrowers are high earners, highly educated, and have large amounts of student debt. The majority (70%) hold more than a bachelor degree, are in their early to mid-30s (IQR = (29, 35)), and earn a post-tax median income of $67,500. Given their graduate degrees, it is not surprising that they also hold large amounts of student debt. The average monthly payment on their refinanced debt, assuming a 10 year fixed rate repayment term, is $600 per month.

The richness of the data allows for a more thorough description of these borrowers beyond monthly income and debt amount. Table 2 shows that 40% are home owners, they spend a median of $1,300 on housing each month, they have an interquartile range of FICO scores ranging from 760 to 800. In terms of assets and liabilities, the median borrower holds $38,000 in assets, $0 in investments (the 75th percentile has $15,000 in investments), $89,000 in liabilities, and has a median monthly free cash flow (post tax income minus student debt and other monthly payments) of $3,100. Borrowers hold a host of degrees and occupations; JDs (lawyers) make up 13% of the sample, MBAs are 17%, MDs 5%, pharmacists 6%, and dentists 4%.

**Loan Terms:** The borrowers’ low risk profile translates into them obtaining considerably lower interest rates when refinancing. Figure 2 shows the distribution of final refinanced APRs plotted against the self-reported distribution of original APRs for a subgroup of the population. The majority of the self-reported original APRs falls within the range of Federal graduate student loan rates (from 6-8%), and the distribution of refinanced APRs is shifted considerably to the left. The distribution of refinanced APRs

---

8The majority of these individuals hold graduate degrees, making them representative of 40% of the $1 trillion student loan portfolio. Graduate students are an important part of the student borrower population, since despite holding the largest amounts of debt (a median of $46,000 amongst 2014 graduates), they have the lowest rates of default and delinquency.
This table contains summary statistics describing the population of borrowers who ultimately refinance their loans. The left columns describe the income and characteristics of the borrowers themselves. Income refers to yearly post-tax income. FCF refers to the monthly post-tax income minus all fixed expenses. Graduate refers to the portion of the population who has a graduate degree. The right columns describe the terms of the refinanced loans. Variable rate is the proportion of loans that have a variable interest rate.

<table>
<thead>
<tr>
<th>Borrower Summary Statistics</th>
<th>Loan Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td><strong>Median</strong></td>
</tr>
<tr>
<td>Income</td>
<td>75,879</td>
</tr>
<tr>
<td>Loan Amt</td>
<td>67,078</td>
</tr>
<tr>
<td>FICO</td>
<td>782</td>
</tr>
<tr>
<td>Mortgages</td>
<td>0.40</td>
</tr>
<tr>
<td>FCF</td>
<td>3,636</td>
</tr>
<tr>
<td>Graduate</td>
<td>0.70</td>
</tr>
<tr>
<td>Age</td>
<td>32.60</td>
</tr>
<tr>
<td>Dependents</td>
<td>0.50</td>
</tr>
</tbody>
</table>

includes variation stemming from risk type, maturity choices, and rate type (fixed vs. variable) choices. The weighted average refiner saved 2.21 percentage points when refinancing, which translated into an average total interest saving of $13,300.

While observable risk determines much of the gains from refinancing, borrowers’ maturity choice also influence how much interest they pay. By extending term, a borrower pays more in total over the life of the loan, but lowers their monthly payment. The full distribution of these total savings relative to monthly payment savings is shown in Figure 2. This scatterplot highlights the fact that many individuals refinancing actually increase their monthly payment in order to further increase total savings. The main mechanism they use to do so is a decreased term. A few individuals also make the tradeoff of increasing total paid in order to further decrease monthly payment - this is achieved through an increased term.

Monthly income is one of the strongest determinants of maturity choice. Figure 3 shows the average monthly payment across the maturity distribution, without controlling for loan size or interest rates. As the maturity increases, the average monthly payment decreases from $1,000 to $400 dollars. However, when rescaled by monthly disposable income, the average DTI ratio stays flat across the maturity distribution –
**Figure 2: Refinancing Differences in Interest Rates, Monthly Payments, and Total Paid**

Notes: The left figure shows the distribution of Annual Percentage Interest Rates (APRs) for the same sample of borrowers before and after refinancing. The self-reported original APR is shown in green – the majority of these interest rates fall in the range of 6-8%, which encompasses the range of typical Federal graduate student interest rates over the past 15 years. APRs reported above this range may represent private student loans that are being refinanced, whereas those below may include undergraduate Federal Loans which have a lower subsidized interest rate. The distribution of new refinanced APRs is more spread out, since risk based prices and term choices generate a larger range of interest rates. The right-hand figure shows how individuals trade off between interest rate savings and monthly payment savings when refinancing. Recall that when individuals choose a longer term, monthly payment decreases but total interest paid increases. Those in the first quadrant are willing to increase their monthly payment to maximize interest rate savings – these individuals are the most interest rate driven borrowers and would have the shortest terms. Individuals in the third quadrant are refinancing primarily to lower their monthly payment, and will actually pay more interest overall. Those in the fourth quadrant are slightly lowering their monthly payment and also lowering their total interest paid.

borrowers seem to target a payment that is 30-40% of monthly income. Over 27% of individuals choose the shortest loan term of 60 months, which reveals a preference for minimizing the total paid in interest.

**Interest Rate Variation:** While borrower-specific characteristics like income and debt level are key determinants of maturity choices, the price of borrowing also plays a role. Holding all else constant, a higher interest rate could induce a shorter (via the substitution effect) or longer (via the income effect) maturity choice.

There are two main types of interest rate variation in my dataset: risk-based and within-risk. In the private sector, the exact variables that determine an individual’s “risk-based” interest rate are proprietary and company-specific. Traditionally, the basis of risk-based pricing formulas has been credit score - for example, the interest rate an individual gets on a mortgage is typically a function of FICO and loan size. However, firms are also able to consider variables like employment, income, liabilities, educa-
Figure 3: Distribution of Maturity Choices and Resulting Monthly Payments

Notes: Both figures plot the histogram of maturity choices for the borrower sample. The left hand figure also plots the average monthly payment for the loans in each maturity bin, while the right hand figure plots the average monthly payment to monthly free cash flow (income minus housing costs) ratio for the loans in each bin. While individuals choosing shorter maturities make larger monthly payments, they tend to have higher monthly FCF. This means that across the maturity spectrum, most borrowers select into a contract that makes them pay 20-40% of their monthly FCF towards their student loan.

Using risk-based variation to identify the elasticity of maturity with respect to interest rates is potentially misleading. Individuals with different risk scores may also differ on unobservable dimensions (like expectations about future income growth or volatility) that will impact their maturity choices. Ideally one would instead use price variation that is orthogonal to all borrower characteristics, including risk score. I refer to this as “within-risk” variation.

The data contains 10 small within-risk score price changes that were conducted at a firm-wide level, and were unrelated to the characteristics of any given borrower. These within-risk price changes moved the entire maturity price schedule up or down for ob-

---

9Legally prohibited risk-based pricing factors under the Equal Credit Opportunity Act are: race, color, religion, national origin, sex, marital status, age, and receipt of income from any public assistance program.

10These non-traditional underwriting techniques have proven popular in the student loan market, where many young borrowers may be credit-worthy but do not necessarily have extensive credit histories.
servationally identical individuals over time (see Figure 4). These price changes created instances when individuals with the same risk profile and observable characteristics, but who applied to refinance at different points in time, would face different interest rate offers.

The price changes were conducted primarily to gather quasi-experimental evidence for the firm on maturity choice and application volume elasticities with respect to interest rates. The price changes occurred over time, not simultaneously for different groups of borrowers, and at a frequency of once to twice a month. This frequency helps alleviate concerns about significant changes in the composition of customers over time (a period of rapid growth), but there are still changes in the observable characteristics of the population over the full set of price changes, addressed below. On average each price change impacted 1,100 borrowers. Borrowers were not aware of the timing of these price changes, and therefore could not respond by adjusting when they refinanced. The changes were not monotonic: interest rates both increased and decreased over time, and therefore will not be confounded by other monotonic trends occurring over time, like company growth.

**Reduced Form Maturity Elasticity**

To estimate a reduced form elasticity of maturity choice to interest rates using the within-risk price variation, I regress each incoming borrower’s initial maturity choice on the interest rates that they were offered, while controlling for observable characteristics. As discussed above, there was variation in the schedules offered to individuals of the same observable risk type over time. Given that each individual faces a continuous menu of maturities and interest rates (as in Figure 4), I include both the 60 and 120 month APR to proxy for the level and slope of the schedule.

I regress the borrower’s maturity choice in months \((T_i)\) on these interest rate variables.
Figure 4: Using Across vs Within Risk Price Variation to Identify Maturity Elasticities

Notes: This figure shows a stylized example of the price variation I use to identify maturity elasticities. The price changes shifted the entire APR/maturity price schedule up and down (i.e. \( r(T) \) vs \( r(T)' \)). Observationally identical individuals were therefore asked to optimize given different budget sets. Panel A illustrates the response of a price inelastic individual – to keep their monthly payment steady, they absorb most of the increase in interest rates. Panel B illustrates the response of a price elastic individual – they decrease their maturity significantly (thereby increasing monthly payment) to keep interest rates near constant.

![Graph](image)

(a) Across Risk Price Variation  
(b) Within Risk Price Variation

as well as a dummied risk score variable,\(^{11}\) the offered variable 120 month interest rate, FICO score, log loan amount, yearly income, monthly free cash flow\(^{12}\), age (dummied), education level (dummied), mortgage number, and number of dependents. Since this specification includes dummies for risk score, it uses only the remaining within-risk price variation to identify the interest rate elasticity.

Table 3 show both the raw coefficients and the coefficients transformed into elasticities. The coefficient on both the interest rate variables has a negative sign – this means that when faced with higher levels of interest rates, similar individuals decrease their loan maturity and increase their monthly payment. Specifically, a 1 percentage point increase in the 120 month APR is associated with a 15.75 month decrease in the average maturity. For the average $70,000 loan with a 108 month maturity, this means that increasing the offered 10 year APR from 5.5% to 5.6% would induce borrowers to increase their monthly payments by approximately 1.7%. This elasticity suggests that

---

\(^{11}\)Individuals at the firm are assigned a numerical risk score on a continuous scale of 1 to 6. In this regression I split the score into .25 unit bins, and control for them as dummies.

\(^{12}\)Monthly free cash flow is defined as monthly income net of taxes and other fixed expenditures (like rent).
### Table 3: Maturity Elasticities (in Months)

<table>
<thead>
<tr>
<th>Coefficients $\frac{dy}{dx}$</th>
<th>Elasticities $\frac{dy}{dxx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed 60</td>
<td>-4.304</td>
</tr>
<tr>
<td></td>
<td>(3.309)</td>
</tr>
<tr>
<td>Fixed 120</td>
<td>-15.75***</td>
</tr>
<tr>
<td></td>
<td>(5.146)</td>
</tr>
<tr>
<td>FICO</td>
<td>-0.0570***</td>
</tr>
<tr>
<td></td>
<td>(0.0147)</td>
</tr>
<tr>
<td>ln(Loan Size)</td>
<td>24.65***</td>
</tr>
<tr>
<td></td>
<td>(0.579)</td>
</tr>
<tr>
<td>Monthly FCF</td>
<td>-0.00319***</td>
</tr>
<tr>
<td></td>
<td>(0.000205)</td>
</tr>
</tbody>
</table>

$N = 10680$

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Standard errors in parentheses

Regression also controls for risk score (dummied), age, variable 120 month rate, FICO, log loan amount, monthly free cash flow, age (dummies), education level, mortgage number, dependents.

---

The average borrower is more sensitive to the interest rate than monthly payments.

The regression also empirically confirms several other economically intuitive relationships – for example, individuals with larger loan amounts and lower monthly incomes systematically choose longer maturities. This suggests that borrowers are making proportional, not absolute, monthly payment decisions with regards to their income level. Individuals with better credit scores choose shorter maturities, even conditional on price, income, and debt amount. This may be driven by the fact that they face fewer credit constraints on other borrowing margins, or are more aggressive in the pay down of their debt.

**Sample Selection Over Price Regimes:** Using temporal price variation presents a selection concern: while some individuals may respond to interest rate changes on the intensive margin by adjusting maturity, others may respond on the extensive margin by no longer refinancing or refinancing with a different company. If those who join or leave the population after a price change have systematically higher or lower maturity preferences, then this extensive margin response will bias intensive margin estimates.

One empirical way to gauge the extent of extensive margin responses is to test whether changes in observable borrower characteristics are correlated with temporal...
variation in interest rates. If the composition of observable characteristics is predicted by the interest rate changes, then there may also be selection on unobservables. Table 4 predicts individuals’ maturity choices, $\hat{T}$, using all observable characteristics other than APR, and then tests whether this variable is predicted by the price regime shifts; these results are insignificant and not large enough to explain the price elasticity estimated in the main specification. Figure 12 graphically shows the lack of correlation between observable characteristics like debt and income and the price shifts.

Table 4: Test of Extensive Margin Response and Changes in Borrower Composition

<table>
<thead>
<tr>
<th>Response of $\hat{T}$ to APR</th>
<th>Coeff.</th>
<th>SE</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>APR *</td>
<td>-3.30</td>
<td>2.433</td>
<td>{-8.07, 1.466}</td>
</tr>
<tr>
<td>N</td>
<td>10680</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 Loan Repayment Model

In this section I outline a stylized model of how borrowers make maturity and refinancing choices. One goal of the borrower model is to be able to interpret the reduced form maturity elasticity in terms of the intertemporal elasticity of substitution.

Borrowers in the model have incurred a fixed amount of student debt while attending school, $D$, have finished with school, and are beginning repayment. In this two-stage model, borrowers choose a repayment maturity, $T$, to maximize their present discounted

\footnote{I model all borrower decisions conditional on debt, schooling, and educational choices, which are made at an earlier period before repayment begins. This equates to the assumption that these decisions are fixed and not impacted by the level of interest rates or ability to refinance debt. This assumption is valid for the population of student borrowers who have already made their loan principal decisions and are yet to make repayment choices (i.e. those currently in school or beginning repayment) – however, it does not apply to individuals who have yet to make borrowing decisions (prospective borrowers who have yet to start school). In a full equilibrium analysis, the level of interest rates and refinancing options could also impact choices like loan principal.}
stream of expected future utility, and whether to refinance their federal loan into the private sector. Two main things distinguish the public and private repayment options: risk-based pricing and maturity-based pricing. The private sector offers interest rates, \( r(T, p) \), that are increasing in a borrower’s observed risk \( p \) and chosen maturity \( T \). The government offers a single price for all risk types and maturities, \( g \). Monthly and total payments could be lower or higher for a given individual in the private vs. public sector - this depends on their risk type and maturity preference.

All borrowers have the same per-period CRRA utility function \( u(c) = \frac{c^{(1-\gamma)}}{(1-\gamma)} \), and discount factor, \( \rho \). While borrowers can control their payment level, they cannot control their variable, growing income \( \tilde{w} \), which I parametrize using a unit root process:

\[
\ln(\tilde{w}_t) = \ln(\tilde{w}_{t-1}) + u_t \\
u_t \sim N(\mu^w, \sigma^2)
\]

where \( \mu^w \) is a yearly growth rate and \( \sigma^2 \) is income variance.

**Step 1: Maturity Demand:** Borrowers of risk type \( p \) and debt amount \( D \) choose a maturity \( T \) to solve:\(^{14}\)

\[
\max_T E\left[ \sum_{t=1}^{T} \beta^t u(w_t - d(T, r)) + \sum_{t=T+1}^{Q} \beta^t u(w_t) \right]
\]

where \( Q \) is the individuals’ maximum age (i.e. finite), \( w_t \) is income plus the maximum debt payment an individual can make,\(^ {15}\) and \( d(T, r) \) is the yearly payment associated with maturity \( T \). Here consumption is defined as income minus the student debt payment \( d(T, r) \) – this implies that individuals are “hand-to-mouth” and not smoothing

\(^{14}\)In the public sector the interest rate \( r(T, p) \) is replaced with \( g \)

\(^{15}\)It is important to define \( w_t \) as yearly income plus the maximum debt payment an individual can make: \( w_t = \tilde{w}_t + d_{MAX} \). This prevents \( w_t - d(T, r) \) from ever being \( \leq 0 \) and causing infinite marginal utility, even in the case of a very small draw of \( \tilde{w} \). Another way of dealing with very negative income shocks is to define a minimum consumption threshold, below which an individual will no longer make a debt payment. I discuss and model this approach in the appendix.
consumption through other means of borrowing or saving.\textsuperscript{16} It also means that changes in maturity translate directly into changes in consumption by impacting the size of the yearly payment, \(d(T, r)\).

The debt payment, \(d(T, r)\), that individuals make each period is a function of their total debt amount \(D\), their chosen maturity, \(T\), and the interest rate schedule that they are offered, \(r(T, p)\):

\[
d(T, r) = D \times \frac{r(T, p)}{(1 - (1 + r(T, p))^{-T})}
\]

As borrowers extend maturity, each periods’ payments become lower (\(\frac{dd(T, r)}{dT} < 0\)), but they pay more over the life of the loan.

Solving the maximization problem results in the first order condition:

\[
0 = -E\left[ \sum_{t=0}^{T} \beta^t \frac{\partial d}{\partial T} u'(w_t - d(T, r)) + \beta^{T+1} u(w(T+1) - d(T, r)) - \beta^{T+1} u(w(T+1)) \right]
\]

This condition says that at the optimal loan maturity, the sum of marginal utility gained from a slightly lower monthly payment is equal to the utility lost from paying additional interest for an extra year.

\textit{The Influence of Interest Rates on Maturity Choice:} The first order condition captures how maturity choices, and therefore utility levels, change with interest rates. All else constant, as the level of interest rates increase, individuals must adjust maturity to maintain the optimality condition. The exact formula for the response \(\frac{dT^*}{dr}\) does not have an analytical solution, but numerical simulations show that as interest rates increase, the optimal maturity choice decreases. As the IES increases, individuals become more interest rate sensitive. This leads to a lower optimal maturity choice at any given interest rate, and also to a steeper relationship between \(T^*\) and \(r\).

\textit{The Influence of Non-Interest Rate Factors on Maturity Choice:} The other non-interest

\textsuperscript{16}I address this assumption empirically in the robustness section.
rate factors in our model that influence maturity demand and interact with the interest rate elasticity are: income level, debt level, income growth and volatility, and the intertemporal elasticity of substitution $\frac{1}{\gamma}$. Due to concave utility, individuals who are low income or high debt gain more marginal utility from decreasing yearly payments, and thus have a higher willingness to pay for long maturities. Individuals who expect income to grow in the future will also prefer a long maturity, since it acts as a means to transfer consumption from the future to the present. Individuals with higher income variability have both higher and less elastic maturity demand due to the fact that longer loans help to smooth consumption across a more variable income profile.

Note that the income-related factors that drive demand (income levels, growth, and volatility) are potentially correlated with risk score $p_i$. Therefore, the same variables that increase demand for maturity on the borrower’s side will also increase interest rates on the supply side. This means that even when faced with higher risk-based prices, high risk borrowers may choose longer loans. This is in line with our reduced form evidence, which showed that, all else constant, riskier borrowers had higher demand for long maturities.

The intertemporal elasticity of substitution ($\frac{1}{\gamma}$) also influences demand – as $\gamma$ increases, an individual will prefer a longer maturity holding all else constant. Intuitively, this is because an individual with concave utility will prefer to smooth consumption by lowering yearly payments, even if it means paying more interest overall. A high level of $\gamma$ (ie a low intertemporal elasticity of substitution) also means that the term choices of individuals will be less responsive to price changes. Thus $\gamma$ is essential for understanding how a borrower’s decisions, and utility, would respond to changes in price.

**Step 2: Refinancing Choice:** Borrowers also decide whether or not to refinance by comparing their level of utility at the optimal maturity across sectors. For very low risk individuals there will be a clear incentive to refinance in the private sector, and for very high risk individuals, there will be no incentive to refinance because government
interest rates will be significantly lower than private rates. For marginal individuals who face a similar level of prices in the private and public sectors, the maturity-based price differential (and whether they prefer shorter or longer loans) could also determine whether they will sort into the private sector. While the model describes the decision to refinance as a discrete choice problem, in reality borrowers might face frictions (inertia, search costs), value non-pecuniary repayment benefits, or have idiosyncratic preferences that prevent them from refinancing even when they would receive lower interest rates.

**Modeling Borrower Delinquency:** This model ignores the impact of delinquency or default on maturity choice, focusing only on the intertemporal consumption tradeoff that comes from a higher or lower monthly payment.\(^\text{17}\) This equates to the assumption that adjusting loan maturity does not impact the probability of delinquency, despite the fact that maturity impacts the size and duration of monthly payments.

One reason for this assumption is that the main model focuses on relationships that can estimated empirically – i.e. the response of maturity to changes in interest rates.\(^\text{18}\) Absent data on delinquency or default it is impossible to directly link maturity choices to repayment outcomes. In the Online Appendix, I instead write down and analyze a model that theoretically links delinquency and maturity choice. In this model, extending maturity will decrease the probability of delinquency, \(P(T)\), in any single period by lowering the monthly payment. It may also increase the probability of delinquency over the life of the loan by extending the repayment period. Misspecification will be more or less of a problem if changes in maturity have a large impact on utility via these delinquency channels. This exercise also provides a robustness test, for various calibrated values, of the central, simplified model which I use in the estimation section. The results suggest that the estimates are not very sensitive to the exclusion of the delinquency

---

\(^{17}\)Currently, borrower income levels and risk impact maturity decisions due to the fact that a low income draw minus a large debt payment will generate a very high marginal utility.

\(^{18}\) Note that the dataset captures only the first 6-36 months of repayment of potentially very lengthy repayment maturities (5-20 years). The absence of delinquency in these initial months does not necessarily mean that delinquency will never occur.
channel – for this low risk group, the first order effect of a change in maturity seems to operate instead through the monthly payment consumption smoothing channel.

5 Estimation of Borrower Model

In this section I estimate the maturity demand model using the same price variation and maturity choices as in the reduced form section. While it imposes stronger assumptions on the borrower’s problem (i.e. calibrating the income process and assuming CRRA utility), it allows us to map the reduced form maturity elasticity to a parameter of economic interest, \( \gamma \), and to ultimately measure changes in consumer surplus. I first discuss estimation of the first order condition using nonlinear least squares, then identification, results, and robustness.

Recall that individuals choose \( T \) to maximize a discounted stream of utility; the resulting first order condition provides our main estimating moment. Most elements of this equation are observed, including: \( T_i \), the optimal maturity choice in months; \( d_i \), the monthly payment for individual \( i \) at term \( T_i \); \( \frac{dd_i}{dT_i} \), the change in monthly payment given a one month increase in maturity from \( T_i \); \( r(T_i, p_i) \), the risk, maturity specific interest rate faced by individual \( i \) at term \( T_i \); and \( w_{i0} \), individuals’ starting level of post-tax monthly income when they first make their maturity choice.

Post-tax income in future periods, \( w_{it} \), is not observed. I assume log income follows a unit root process and calibrate both the growth rate and volatility.\(^{19}\) The calibration details are provided below – I predict the growth rate, \( \hat{\mu}_w \), using a separate cross-sectional dataset of personal loan applicants, and calibrate income volatility, \( \hat{\sigma}_I^2 \), using the repayment probabilities implied by a simple lending model.

The remaining parameter left to estimate is the intertemporal elasticity of substitution, which I model as a function of observable characteristics (\( X_i \)) including degree

\(^{19}\)In the robustness checks I relax this assumption and try other specifications.
type, risk score, current disposable income, student loan amount, age, FICO score, home ownership, and number of dependents:

\[
\gamma_i = \gamma + X_i'\mu + \epsilon_i
\]

Allowing \( \gamma_i \) to vary with \( X_i \) will control for changes in observable characteristics across price regimes. Following the logic outlined in the reduced form section, it is also important to include observed risk score \( (p_i) \) directly in the model. If \( p_i \) was not included in the estimation, the model would wrongly attribute differences in maturity choices across risk type to differences in offered APR, and our estimate of \( \gamma \) would suffer from omitted variable bias.

In the main specification, I use a certainty equivalence approach to write the first order condition as a closed form analytical expression. I rewrite the expected marginal utility as the marginal utility of a certainty equivalent given by:

\[
E[(w_{i0} * e^{t*p_i} * e^{\sum_i u_{it} - d_i})^{-\gamma_i}] = (w_{i0} * e^{t*p_i} * e^{\pi_{it} - d_i})^{-\gamma_i}
\]

where \( \pi_{it} \) is the certain amount an individual would have to be given in that period to make the certainty equivalent equal to the expected marginal utility. As income volatility, \( \gamma \), and the debt to income ratio \( (w_{i0} * e^{t*p_i} - d_i) \) ratio increase, the certainty equivalent becomes more negative. Specifically\(^20\):

\[
\pi_{it} = \frac{1}{2} * t * \hat{\sigma}_t^2[1 - (1 + \gamma_i)\frac{w_{i0} * e^{t*p_i}}{w_{i0} * e^{t*p_i} - d_i}] \quad \text{for} \ t < T+1
\]

\[
\pi_{it} = \frac{1}{2} * t * \hat{\sigma}_t^2(-\gamma_i) \quad \text{for} \ t \geq T+1
\]

I observe individuals making a maturity choice on a monthly level. I approximate \( \frac{\Delta d_i}{\Delta T} \approx \)

\(^20\)For the exact derivation of \( \pi_{it} \) see the Online Appendix
\[ \frac{\partial d_i}{\partial T_i} \] as the change in monthly payment given a month change in maturity.\(^{21}\) With these changes, the analytical estimating moment becomes:\(^{22}\)

\[
h_i(\theta) = \sum_{1}^{T_i} \beta^t \frac{\Delta d_i}{\Delta T_i} \left( w_{i0} * e^{t*\hat{w}_i} * e^{\hat{\mu}_i} - d_i \right)^{-\gamma_i} - \beta^{T_i+1} \left( -d_i \right) \left( w_{i0} * e^{(T_i+1)*\hat{w}_i} * e^{\hat{\mu}_i(T_i+1)} \right)^{-\gamma_i}
\]

This makes the first order condition a nonlinear function of observable variables, \((r(T_i, p_i), T_i, w_{i0}, D_i, X_i, \hat{\mu}_i, \hat{\sigma}_i^2)\), and unobservable parameters, \(\theta = \{\gamma, \mu\}\), that I estimate using nonlinear least squares. I calibrate a yearly discount rate of .98.

**Calibration of Income Volatility and Growth Rates**

Estimation of the borrower problem requires calibration of income growth and volatility. Here I explain how I derive implied income risk from a simple lending model and estimate income growth rates.

**Lending Model:** I assume that lenders in this sector are perfectly competitive, and therefore set interest rates such that they are indifferent between lending to a risky borrower at interest rate \(r(p, T)\), and lending to a risk-free borrower at interest rate \(i(T)\).

I assume that the risk-free rate \(i(T)\) also incorporates the fixed costs that the company must incur when lending (i.e. origination costs and cost of capital), and therefore the only difference between \(i(T)\) and \(r(p, T)\) is the repayment risk premium.

I assume borrowers will become delinquent on a loan in a given period if income falls below a certain threshold – i.e. \(w_i - d(T_i) < x\).\(^{23}\) This means that the probability

---

\(^{21}\)Specifically, \(\frac{\Delta d_i}{\Delta T_i} = d_i(T_i + 1) - d_i(T_i)\)

\(^{22}\)To make this condition empirically tractable, I replace the second term, \(u(w_{(T+1)} - d(T, r)) - u(w_{(T+1)} - d(T, r))\), with the mid-point approximation \(-d(T, r) * u'(w_{(T+1)}) - d(T, r)/2\).

\(^{23}\)A model that also incorporates the delinquency option into the borrower demand model is provided in the Online Appendix.
that a borrower is delinquent in a given period is:

\[ P(T_i) = \frac{\Phi(ln(x + d(T_i)) - ln(w_i))}{\sigma_i} \]  

(1)

Lenders have a recovery rate of \( \alpha \) if a payment is delinquent. Given the delinquency probability and recovery rate, a lender’s expected stream of payments on a loan of maturity \( T \) to risk type \( p \) is:

\[ T * \left( (1 - P(T)) * D * \frac{r(T, p)}{(1 - (1 + r(T, p))^{-T})} + \alpha * P(T) * D * \frac{r(T, p)}{(1 - (1 + r(T, p))^{-T})} \right) \]

while the expected stream of payments on a risk-free loan of maturity \( T \) is:

\[ T * \left( D * \frac{i(T)}{(1 - (1 + i(T))^{-T})} \right) \]

Lenders set \( r(p, T) \) to satisfy the indifference condition:

\[ P(T) = \left( \frac{i(T)}{(1 - (1 + i(T))^{-T})} * \left( 1 - \frac{1}{r(T, p)} \right) - 1 \right) * \frac{1}{(\alpha - 1)} \]  

(2)

I observe everything on the right-hand side of this equation, which will allow me to calculate empirical delinquency probabilities for every individual in the dataset, which I call \( \hat{P}(T) \). Combining \( \hat{P}(T) \) with equation (1) allows me to then calculate individual specific \( \hat{\sigma}_i \):

\[ \hat{\sigma}_i = \frac{\Phi(ln(x + d(T_i)) - ln(w_i))}{\hat{P}(T_i)} \]

**Calibration of Income Volatility:** I directly observe the empirical values of \( r(T, p) \) and \( T \) for each borrower, and infer values for \( \alpha \) and \( i(T) \) from the firms’ cost accounting documentation. Table 8 lists the summary statistics for these variables. The calibrated recovery rate \( \alpha = .15 \) reflects the model’s per-payment recovery assumption – i.e. if a
borrower misses a single payment, the lender will recover $\alpha$ of that period’s payment, and then continue to collect the remaining loan capital at the full rate. In reality, the lender often charges-off the entire loan to a collections agency, which means they may have a higher average recovery rate that applies to the entire loan principal, but a very low recovery rate in any single period (the model’s $\alpha$).

The “risk-free” rate of lending, $i(T)$, captures various costs of lending for the firm at that maturity, including the cost of capital, the cost of customer acquisition, and the cost of servicing the loan. In fact, the only cost not included in this measure is the expected cost of non-repayment (i.e. delinquency). Therefore, $i(T)$ is the same for borrowers of all risk types, but does vary with the maturity of the loan. In contrast, $r(T)$ varies with both the risk type of the borrower and the maturity of the loan. Equation 2 and these values produce estimates of $\hat{P}(T)$ in the range of .003 to .15 (see Figure 11 for a histogram of the calculated probabilities). For context, historical yearly delinquency and default rates for graduate students in the government’s Direct Loan portfolio range from 3-7%.

I next transition to the borrower problem, and calculate the individual-specific income volatility that would generate the estimated values of $\hat{P}(T)$. This relies on Equation 1, which, under the assumption that income is log-normally distributed, maps $\hat{P}(T_i)$ to $\hat{\sigma}_i$. For these calculations, I use post-tax income as an empirical measure of $w_0$, and individuals’ verified housing expenditures as a proxy for $x$, the minimum consumption threshold. This measure of $x$ assumes that individuals will first pay their rent or mortgage before making their student debt payment. As $x$ becomes larger relative to $w_0$, $\sigma_i$ will become smaller: a smaller shock to income is needed to push an individual into delinquency.

The resulting income variance estimates are described in the final row of table 8. The median value of $\sigma_i$ is .29, off a median value of $\ln(w)$ of 11.16. Interpreted for a median individual, these estimates imply that someone initially making $50,000 has a
68% chance of making anywhere between $40,000 and $82,000 in each of the following years.

**Calibration of Income Growth Rates:** While one would ideally use panel data to directly observe the income growth of my borrowers, the long time horizon of the debt contracts (up to 20 years) is a limiting factor. I instead use a cross section of observationally similar individuals at various ages to estimate a pseudo age-income profile.

The dataset I use to estimate these cross-sectional profiles contains individuals who are similar to my refinancing applicants in many important respects (high income, high FICO, mainly graduate degree recipients), but who are applying instead for small personal loans rather than applying to refinance student debt. This distinction is important when estimating cross-sectional age profiles – if I instead used a cross-section from the student loan borrower population, one might worry that individuals refinancing student debt at age 40 have very different income trajectories than those refinancing at age 30. Here the worry is that individuals borrowing small amounts ($5,000 - $15,000) at different ages have fundamentally different earning trajectories. While this selection concern is valid, one must weigh it against the fact that this population is similar to my borrowers in many unique respects that would be difficult to find and match to in a survey dataset.

For example, my population is refinancing with a new internet-based bank, which makes them potentially different than a population that uses only traditional banks. Furthermore, because my sample has a high socioeconomic status, they make up only a small percentage of most representative survey samples.

This dataset contains approximately 250,000 borrowers who range in age from 20 to 50. Figure 5 gives a sense of what the cross-sectional income trajectories look like for individuals in this sample – it plots monthly post-tax income after separating individuals into 4 degree levels: associates, bachelors, masters, and professional. It also compares the growth rates and levels to those found in the CPS – in large part the two datasets seem to capture similar earnings trajectories, at least at the level of degree type.
Figure 5: Monthly Post-Tax Income, by Age and Degree

Here I plot a comparison of monthly post-tax income for individuals with different degree types and occupations within the dataset I use for estimation and the CPS. The personal loan data that I use for estimation seems to reflect a similar growth pattern to that found in the CPS, in particular for Associates, Bachelors, and Master degree recipients. Professional degree holders in the CPS data have a slightly higher level and steeper growth rate of monthly post tax income.

![Graph](image)

(a) Personal Loan Dataset (used for estimation)  
(b) CPS Dataset

Using this sample, I estimate degree-specific growth rates that match the log-income parametrization of my model with the following regression:

\[
\ln(w_i) = \beta_0 + \beta_1 \ast \text{age}_i + \gamma_0 \ast \text{degree}_i + \gamma_1 \ast \text{age}_i \ast \text{degree}_i + e_i
\]

Here \(\ln(w_i)\) is log post-tax yearly income, and \(\text{degree}_i\) are dummies indicating highest degree level. The coefficient \(\hat{\beta}_1\) estimates the average yearly growth rate of log income over the life-cycle, while \(\hat{\gamma}_1\) allows this growth rate to deviate by degree type. The estimated growth rate for any borrower in my student loan dataset is thus given by \(\hat{g}_i = (\hat{\beta}_1 + \hat{\gamma}_1)\).

Empirical Identification

The identification intuition for the structural model remains very similar to that in the reduced form section: there, maturity choice was expressed as a linear function

\[\text{Empirical Identification}

24While we would like to estimate \(g_i\) that vary by many characteristics, from number of dependents to FICO score, the use of cross-sectional data only allows for comparison on \textit{time-invariant} characteristics like degree type, or occupation. Gender would be another relevant characteristic to include in this regression, but unfortunately it is not recorded in the dataset.
of observables, risk type, and interest rate \((X_i, p_i, \text{and } r(T, p)_i)\), with the interest rate coefficient \((\beta)\) identified using the series of price changes that were orthogonal to \(p_i\).

In the structural estimation, \(T_i\) is instead expressed as an implicit non-linear function of \(X_i, p_i, \text{and } r(T, p)_i\), derived from the borrower’s first order condition, and the price changes now serve to identify \(\gamma\).

In both models, the price changes provide moment conditions in which observationally identical individuals (in terms of both \(X_i\) and \(p_i\)) face a different set of interest rates \((r \text{ vs } r’)\) and make potentially different maturity decisions. The parameters, \(\beta\) and \(\gamma\), must rationalize how changes in maturity decisions respond to changes in interest rates. They are therefore estimated using shifts in the maturity distribution over price regimes, and not the static maturity distribution itself. Using this price variation requires the assumption that conditional on \(X_i\) and \(p_i\), any unobserved characteristics of borrowers across price regimes are uncorrelated with their maturity choices.

One key difference between the two specifications is that income growth and volatility are directly modeled in the structural section. In the reduced form analysis, income growth and volatility are one of many sources of unobservable heterogeneity that might impact maturity choice – they are left in the error term, and assumed to be orthogonal to the price changes. In the structural analysis, income growth and volatility become a source of observable heterogeneity. By modeling the income path over time and over states of nature, we can translate changes in maturity into changes in consumption. While this puts more structure on the borrowers’ problem, it also allows us to interpret the reduced form maturity elasticity as a more economically relevant parameter, the IES.

In the model, \(\gamma\) plays the role of both the IES and the risk aversion parameter. Empirically, the parameter is identified from a consumption smoothing decision – how changes in monthly payment respond to changes in interest rates – and thus should be interpreted as the IES. In the counterfactual, changes in welfare also come from intertemporal changes in consumption, not changes in income risk.
A final concern is to correctly attribute what changes in term choice across price regimes comes from actual changes in interest rate, and what portion comes from changes in sample composition. Again, modeling $\gamma$ as a function of $X_i$ and $p_i$ will control for the impact of observable heterogeneity across price regimes on maturity choice.

Results

The results from the structural estimation are shown in Table 5. The first column estimates come from our preferred specification, which models $\gamma$ as a function observable characteristics, and the remaining columns report results from specifications with alternative assumptions, discussed in more detail below.

The average estimate of $\gamma$ in the primary, and in all specifications, falls on the moderate to low end of estimates in the existing literature. It translates into an IES of .55, and most recent micro estimates have found a IES from .2-.6. This value implies that on average there is a sizable consumption response to changes in interest rates. The small estimate of $\gamma$ is not surprising in light of our sample, setting, and earlier results. It is reflected in both the distribution of term choices, in which one quarter of borrowers choose a maturity below 6 years, and the reduced form results, which found a relatively large maturity elasticity.

It is also possible that the type of debt studied here could also have a unique psychological impact on estimates of $\gamma$. The amount of student debt a borrower has is more often determined by “necessity” (due to the level of tuition or financial aid available at the individual’s school), rather than choice and thus be perceived as more burdensome and unwanted. Therefore borrowers might treat their student loans differently than other forms of debt or savings, and want to pay it off more quickly. These results underscore the importance of considering several models of consumer behavior when analyzing saving and borrowing decisions - while our lifecycle model of repayment rationalizes these choices under the assumption of full information and rational expectations, there
may in fact be behavioral tendencies, for example debt aversion or rule of thumb accounting, that are driving some portion of individuals’ behavior.

The main specification estimates $\gamma$ as a function of observables, returning a distribution of predicted values from 1.2 to 2.6. As explained in the identification section, this helps isolate the exogenous price changes as the main source of identifying variation for the intertemporal elasticity of substitution, but it also allows us to compare the IES across different characteristics. There is a lower elasticity amongst older individuals, those with larger families, and those with lower credit scores. Perhaps more surprising is that individuals with higher income, lower amounts of student debt, and lower risk scores have a lower IES, which seems to contradict the reduced form results. However, heterogeneity in the IES estimates should not necessarily map 1-to-1 to the reduced form findings: the structural model allows for much more complexity than our OLS analysis, as it captures, for example, the curvature of the monthly payment tradeoff at different points in the maturity distribution, the full interest rate schedule, or the role of expected income growth. A high income individual may be making a much smaller consumption tradeoff when they reduce maturity by one month than a low income individual making the same decision. Therefore, even though low risk or high income individuals may have a higher reduced form maturity elasticity, when these additional non-linear factors are considered, they actually have a lower IES.

Robustness Analyses

In this section I test the robustness of my modeling assumptions in several ways: first, I test how sensitive estimates of $\gamma$ are to the inclusion or exclusion of delinquency in the borrower’s problem. Second, I examine whether borrowers are adjusting on other financial margins, to see if changes in maturity truly represent changes in consumption. Finally, I test the modeling assumption that borrowers are making a single, not recurrent, maturity choice by looking at prepayment rates in the data.
Table 5: Maturity Choice Model Estimates

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>(1) Main Specification - $\gamma_i = f(X_i)$</th>
<th>(2) Homogenous $\gamma$</th>
<th>(3) No Income Risk</th>
<th>(4) No Income Growth or Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ - Constant</td>
<td>1.704 (0.269)</td>
<td>1.416 (0.01)</td>
<td>5.900 (1.771)</td>
<td>8.369 (3.122)</td>
</tr>
<tr>
<td>$\gamma$ - log(Income)</td>
<td>0.203 (0.023)</td>
<td>2.356 (0.130)</td>
<td>3.269 (0.247)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ - log(Debt)</td>
<td>-0.234 (0.010)</td>
<td>-2.361 (0.064)</td>
<td>-3.434 (0.147)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ - Home Owner</td>
<td>-0.0532 (0.0126)</td>
<td>0.1689 (0.0800)</td>
<td>0.1484 (0.1328)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ - # Dependents</td>
<td>3.76E-02 (8.60E-03)</td>
<td>2.56E-01 (4.83E-02)</td>
<td>3.50E-01 (8.44E-02)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ - Age</td>
<td>6.10E-03 (1.10E-03)</td>
<td>2.40E-02 (7.15E-03)</td>
<td>3.08E-02 (1.31E-02)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ - Risk Score</td>
<td>0.124 (0.007)</td>
<td>-0.455 (0.047)</td>
<td>-0.464 (0.081)</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ - FICO</td>
<td>-3.00E-04 (2.00E-04)</td>
<td>-1.98E-03 (1.14E-03)</td>
<td>-2.15E-03 (1.97E-03)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{60}$</td>
<td>2.477 (0.315)</td>
<td>-0.578 (0.38)</td>
<td>4.219 (0.349)</td>
<td>4.691 (0.424)</td>
</tr>
<tr>
<td>$\alpha_{240}$</td>
<td>-71.695 (1.992)</td>
<td>-72.00 (2.06)</td>
<td>-75.043 (2.149)</td>
<td>-84.091 (2.400)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. This table presents results from the non-linear least squares estimates of the borrower’s maturity choice model. Income is defined as yearly post-tax income. Debt is the amount of student loan debt the individual is refinancing. Home Owner is a dummy indicating whether an individual owns a home. Risk Score is the firm specific score that is used as the basis for risk based prices; a higher score indicates lower risk. $\alpha_i(x)$ are dummy variables that indicate whether an individual chose the minimum or maximum maturity to account for the truncation of the maturity choice set at 5 and 20 years.

Column (1) presents the main specification, in which the intertemporal elasticity of substitution parameter ($\gamma_i$) is modeled as a function of observables; Column (2) does not allow $\gamma_i$ to vary with observables. In both (1) and (2) the future income growth and volatility of the borrowers are calibrated using an external data source (see the appendix for details). In (3) I only calibrate income growth, and make income deterministic (not risky). This increases the estimate of $\gamma$, since without income volatility individuals must have a lower IES to rationalize the same maturity choices in the data. In (4) I remove income growth (income stays constant) and income volatility. Again, this increases the estimate of $\gamma$.33
Impact of Delinquency on Estimates: While the estimated model abstracts away from delinquency, in the Online Appendix I write down a model that includes delinquency and adds additional terms to the first order condition. Here I calculate what happens to \( \hat{\gamma} \) as we increase or decrease the magnitude of the additional terms.

The first order condition with delinquency includes an unobservable utility “penalty”, \( G \), that a borrower would face if they ever missed a payment on their loan.\(^{25}\) Figure 6, Panel 2 tests how \( \gamma \) changes across a broad range of values of \( G \), from a penalty that would reduce utility by 0% for the next 50 years, all the way to 70%. The value of \( G \) implied by our estimate of \( \hat{\gamma} \) is relatively low, suggesting our estimate may be biased downwards. But even across a wider domain of penalty values, the implied range of \( \gamma \) is relatively narrow.

I also test how a range of delinquency probabilities, \( P(T) = .001 \) to .3, would impact an estimate of \( \gamma \). Again, the value of \( \gamma \) does not change considerably, moving from 1.6 to 2. The insensitivity of \( \gamma \) to these additional terms makes some intuitive sense. For borrowers in our dataset who are relatively low risk, the derivative of the probability of delinquency with respect to maturity is small. This means that the first order impact of a change in maturity on utility comes from the change in monthly payment, which we capture in the main specification, not from the change in the probability of delinquency.

Contemporaneous Financial Decisions: Identification of \( \gamma \) hinges on the fact that quasi-experimental variation in interest rates caused otherwise identical individuals to choose slightly different monthly payments, \( d(T) \). The hand-to-mouth assumption in the model maps these changes in \( d(T) \) directly to changes in consumption. In reality individuals may be saving, rather than consuming, this residual, or adjusting on other borrowing margins. Therefore, we want to test whether there was a differential change in the saving or borrowing rate before and after refinancing for individuals who faced slightly different interest rate schedules.

\(^{25}\)This was one obstacle to estimating the delinquency model: while the lender’s problem provides observable delinquency probabilities, there is no clear empirical proxy for \( G \).
Figure 6: Comparison of $\gamma$ under delinquency and non-delinquency model.

Notes: This figure (right panel) shows what happens to estimates of $\gamma$ when the model allows for delinquency, which adds an unobservable utility “penalty”, $G$, that a borrower would face if they ever missed a payment on their loan. Across a wide domain of penalty values, the implied range of $\gamma$ is relatively narrow. I also test how a range of delinquency probabilities, $P(T) = .001$ to $.3$, would impact an estimate of $\gamma$ (left panel). Again, the value of $\gamma$ does not change considerably, moving from 1.6 to 2.

I directly observe the outstanding debt and debt payment of borrowers in my sample, which are substantial. Over 40% of borrowers have a mortgage, which on average translates into a $1,900 payment. All of the borrowers have some sort of fixed monthly payment on their credit reports: 40% of borrowers have monthly auto payments which are on average $450, 75% have credit card payments, and 90% have uncategorized installment debt.

I can test the hand-to-mouth assumption by measuring whether borrowers’ monthly payments adjust immediately before and after refinancing. Table 14 in the appendix describes changes in other monthly payments (mortgages, auto loans, credit cards, etc) before vs. after refinancing for individuals who had positive monthly payments to begin with, and shows that for the vast majority of borrowers these stayed constant. This is perhaps because many of these payments are fixed installments, and it would take active work on the borrower’s part to readjust them.

While I do not observe savings rates before and after refinancing, I do observe the
level of savings of borrowers in my sample. Slightly under 40% have a formal retirement savings account – for example 25% have a 401k, with a median balance of $24,000. The number of individuals with investment holdings increases with age. Figure 15 shows that while the median borrower continues to not have substantial savings through age 60, the 75th percentile has accumulated over $80,000 by age 50. However, 90% of my borrowers are under 40 years old, and therefore even the most active savers have investment holdings that are much smaller than their student debt amount.

**Evidence on Permanence of Term Choice:** Our model assumes that borrowers make a maturity choice in year 1 to maximize expected utility over the life of the loan. One might question whether borrowers are actually optimizing over such a long time horizon, or if they are in fact choosing a monthly payment to fit their current income level, with the intent to refinance and change term yet again in the future when their income level changes.

To address this, I look at payment patterns over time within my sample of refinancers – in other words, do any individuals keep their payment level over time constant, or do they systematically make higher or lower payments on their debt. I find that there are some extra payments in the data, but they are small and do not vary systematically over time. Figure 13 in the Appendix shows that each month borrowers pay on average 1.5% more than their regular payment, and this is driven by on average only 1% of borrowers making a extra payment each month. There is also no systematic trend in the extra payments. One might expect payments to increase with time as income increases, but here the level of extra payments stays constant over the two year period.

### 6 Welfare Analysis

In this section, I use the estimated demand model to analyze how advances in private sector risk-based pricing impact the size and distribution of consumer surplus. My
benchmark for these comparisons pools all borrowers (high and low risk) under a uniform interest rate, which represents how the federal loan program would operate with no private refinancing option. I measure the extent of cross-subsidization generated under uniform pricing across risk types and income levels, as well as the deadweight loss. I next introduce a private refinancing option with varying degrees of pricing sophistication, from very coarse (FICO score) to fine grained (our risk score $p_i$), and measure two main effects: the net increase in consumer surplus, as low risk types refinance into lower, more efficient risk-based prices, and the increase in average cost for the federal program as low risk types select into the private market.

For these exercises, I use the sample of all refinancing applicants – individuals who received a refinancing price quote, but who did not necessarily complete the entire refinancing process. This is different from my estimation sample, which included only approved, agreed refinancers. The applicant sample is more representative of the federal loan portfolio, but the exercise requires that I extrapolate my estimates to a group with a much wider distribution of income, FICO score, and debt amount. To limit the extent of the extrapolation, I restrict the applicant sample to individuals who have a debt-to-income ratio that overlaps with the support of the refinancing sample.

**Supply Side Assumptions:** In the private refinancing sector, I assume that the risk-based interest rates that firms offer ($r(T, p_i)$) are equal to the expected costs of lending to individual $i$ over maturity $T$. This equates to the assumption of a perfectly competitive refinancing market - i.e. if a firm charged a mark-up, another firm could enter the market and offer a slightly lower price to the same individual while still breaking even. The refinancing market displays most features of perfect competition, including rapid entry into the industry by many firms, and little product differentiation. It is very easy for consumers to price shop and compare price quotes online across refinancing firms. I also estimate very high elasticities (larger than 5) to refinance with respect to offered interest rates in the data, which suggests that price competition across refinancing firms
is robust. Importantly, this assumption means that there will be no changes in producer surplus \((PS = 0)\) during the counterfactual, only consumer surplus.

For the federal loan program, I use risk-based discount rates to estimate the size of each per-borrower subsidy under a uniform interest rate regime – specifically, I discount future cash flows under the uniform price regime with the risk, maturity-specific interest rates that would be assigned to that loan in the private sector.\(^{26}\) The risk-adjusted stream of cash flows, where monthly payments under uniform pricing are given by \(d_i(g)\) and term choice is \(T\), is:

\[
PRDV_i(g) = \frac{d_i(g)}{r(T, p_i)} \left[1 - \frac{1}{(1 + r(T, p_i))^T}\right]
\]

The value of the subsidy is given by the difference between the risk-adjusted present value of the loan, and the loan principal (which is equivalent to the present value of the loan without risk adjustment). The subsidy is positive for high risk borrowers and negative for low risk borrowers. If the government were to conduct revenue-neutral pricing, the breakeven interest rate \(g\) would be defined by:

\[
\tilde{g} = \left\{ g : \sum_{i=1}^{N} D_i - \sum_{i=1}^{N} PRDV_i(g) = 0 \right\}
\]

**Baseline: Fully Uniform Pricing**

As a benchmark for my analysis, I assume all individuals in my sample are forced into a uniform pricing scheme at a rate that is revenue neutral. Uniform pricing will generate deadweight loss as some individuals choose maturities at a price that is above or below the expected cost of providing to them - the low risk types end up choosing shorter loans than they would in a setting where they are charged the cost of providing

\(^{26}\)By using these market prices, I focus only on risk that is observable and priced in the private sector but unpriced in the public sector. I also assume that expected losses given risk type and term are the same in the private and public sector. This assumption seems reasonable given that both sector treat default and delinquency similarly.
Figure 7: Equity and Efficiency Impact of Uniform vs. Risk-based Pricing

Notes: This figure depicts the maturity choices a high and low risk borrower would make in the private sector and the public sector. There are two maturity demand curves, the lower one for the low risk borrower, and the higher one for a high risk borrower. In the private sector they face the two risk specific price schedules, \( r(p_H) \) and \( r(p_L) \), and choose terms \( T^*_H \) and \( T^*_L \), which are efficient. In the public sector, they instead both face the uniform price \( g \). The low risk type chooses a much shorter term \( T_{G,L} \), and the high risk type chooses a much longer term \( T_{G,H} \) than in the private sector. Implicitly, under the uniform rate \( g \), the low risk type is being taxed while the high risk type is receiving a transfer.

Using our model, we can quantify both the deadweight loss and the redistribution represented here graphically. The breakeven interest rate for this sample is \( g = 6.4\% \), which is in the range of existing Federal Interest Rates for graduate students. To calculate the deadweight loss associated with the uniform interest rate, I calculate the transfer, in addition to the revenue collected, that would make each individual indifferent between a uniform and risk-based pricing regime.\(^{27}\) Simply returning the revenue to the borrower is insufficient compensation, since the maturity elasticity (i.e. the substitution...

\(^{27}\)This requires predicting individual’s optimal maturity choice under each pricing scenario using the calibrated demand model, and calculating the size of the subsidy or tax they face at that maturity under the uniform rate.
effect) is nonzero. On average, the per borrower DLW due to the maturity distortion is $448, or 32% of the average tax/transfer. These calculations suggest that a uniform interest rate is a relatively inefficient means of redistribution, which is unsurprising in light of the high estimated IES and maturity elasticity.

Redistribution occurs primarily over risk type, given that risk type directly determines an individual’s true “price” and therefore the size of the implicit tax or subsidy they face. On average, individuals who are low risk are taxed $1,082 under uniform pricing (relative to risk-based pricing), whereas individuals who are high risk (and thus face lower interest rates under uniform pricing) gain an average of $1,507. The redistribution achieved over a more equity-relevant variable, income, is modest. Figure 8 plots the average subsidy given to each borrower under uniform pricing over both borrower risk type and borrower income. The lowest income borrowers get a subsidy of slightly more than $1,000, while the riskiest borrowers get an average subsidy of almost $3,000. This is because income is not perfectly correlated with risk type or maturity preferences (the two dimensions that differentiate costs and thus directly generate redistribution), and therefore the uniform rate is an imperfect instrument for achieving redistribution over income.

Counterfactual I: Innovations in Risk-Based Pricing and Expansion of the Private Refinancing Market

I next analyze what happens to sorting and welfare in the market as risk-based pricing technologies advance and refinancing firms are able to price on more characteristics. Panel (a) of Figure 9 shows how innovations in risk-based pricing increase the distribution of interest rates charged in the private sector relative to a more coarse measure of borrower quality like FICO score. Here I calculate and plot the 10 year fixed interest rate each borrower would face if the firm could only price on FICO score, as well as the 10 year fixed interest rate each borrower would face at the current “frontier” of risk-based
Figure 8: Redistribution over Risk and Income under a Uniform Interest Rate

Notes: This figure plots the average per borrower tax or transfer under a uniform interest rate policy for 20 income quantiles and 20 risk type quantiles. While the redistribution from lowest to highest risk quantile is large (over $3,000 on average per borrower), less redistribution occurs from lowest to highest income quantile.

The graph shows that more comprehensive risk-based pricing expands the distribution of interest rates, in particular extending lower interest rates to the least risky types. The gains to considering additional characteristics are especially large for the student borrower population: because these borrowers are young and have less-developed FICO scores, this allows them to signal their risk type through other characteristics like degree type, savings behavior, or income.

Using these prices, we can calculate changes in borrower surplus as low risk individuals refinance out of the public sector to take advantage of lower rates. In this initial analysis I assume all individuals who would benefit from refinancing do so. Table 6, Column 1, displays the average gain from risk-based pricing, defined as the change in present discounted cash flows, for individuals who refinance. By leaving the uniform

\[ r(T, p_i) \]

I use the observed interest rates schedules from my dataset as empirical proxies for \( r(T, p_i) \). The firm estimates borrower risk using a predictive algorithm to estimate the probability of delinquency. This algorithm produces a risk score \( p_i \) for each individual based on a vector of characteristics, \( X_i \). This score maps to a schedule of risk, maturity specific interest rates \( r(T, p_i) \).
regime, these consumers gain on average $1,082. Using comprehensive risk-based pricing, rather than FICO-based pricing, increases these gains substantially by $341 per borrower.

From the government’s perspective, when these low-risk individuals leave the break-even program to refinance, the average risk of the remaining pool increases. Under the assumption that the government maintains the same interest rate even when low risk types exit, I calculate the average per borrower subsidy for the remaining set of borrowers in column 2. The exit of low risk types under full risk-based pricing increases the subsidy from $0 to $1,507. While innovations in risk-based pricing increased the gains for individuals who refinanced relative to FICO-based pricing, the two pricing schemes generate smaller changes in the new subsidy, only a $247 value. This is because innovations in risk-based pricing do not increase the extensive number of refinance, but rather extend lower interest rates to individuals who would have already benefited.
Table 6: Impact of Pricing and Refinancing on Surplus and Revenue

<table>
<thead>
<tr>
<th></th>
<th>Average Tax*</th>
<th>Average Subsidy**</th>
<th>Average DWL***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Pooling, Break-even $g$</td>
<td>1082.52</td>
<td>1507.48</td>
<td>448.19</td>
</tr>
<tr>
<td>Complete FICO-based pricing</td>
<td>580.99</td>
<td>745.80</td>
<td>13.06</td>
</tr>
<tr>
<td>Complete Risk-based pricing</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Average Changes in Surplus as Individuals Refinance Out of Break-even Pool

<table>
<thead>
<tr>
<th></th>
<th>Avg. $\Delta PRDV_i$ for Refinancers†</th>
<th>Avg. $\Delta$ Subsidy††</th>
<th>Avg. $\Delta$ DWL†††</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refinance into FICO-based Prices</td>
<td>741.20</td>
<td>1259.80</td>
<td>-174.29</td>
</tr>
<tr>
<td>Refinance into Full Risk-based Prices</td>
<td>1082.52</td>
<td>1507.48</td>
<td>-221.87</td>
</tr>
</tbody>
</table>

* Average Tax defined as the average negative change in present risk discounted value relative to the risk-based price regime. For government pooling this change is defined as $D_i - PRDV_i(g)$. For FICO-based pricing this is defined as $D_i - PRDV_i(r(FICO_i))$. The present value is risk adjusted using the risk-based interest rates $r(p_i)$, which means that $PRDV_i(r(p_i)) = D_i$.

** Average Subsidy defined as the average positive change in present risk discounted value relative to the risk-based price regime.

*** DWL is calculated as the transfer, in addition to the tax or subsidy collected, that would make each individual indifferent between $g$ or $r(FICO_i)$ and $r(p_i)$.

† Average $\Delta PRDV_i$ for refinancers is defined as $PRDV_i(r(FICO_i)) - PRDV_i(g)$ for individuals refinancing under a FICO based regime. For individuals refinancing under a fully risk based price regime, $\Delta PRDV_i = PRDV_i(r(p_i)) - PRDV_i(g) = D_i - PRDV_i(g)$ = average tax under full pooling.

†† Avg. $\Delta$ Subsidy is defined as the new average subsidy the government will be providing under their original breakeven interest rate for individuals who do not refinance into the private sector.

††† $\Delta$ DWL is defined as $DWL(r) - DWL(g)$. There will be no change in DWL for individuals who do not refinance – $DWL(g) - DWL(g) = 0$. There will be a decrease in DWL for individuals who refinance, but the change will be smaller for individuals refinancing into FICO based prices – $|DWL(r(FICO_i)) - DWL(g)| < |DWL(r(p_i)) - DWL(g)| = DWL(g)$.

Finally, we can calculate the decrease in deadweight loss that occurs as accurate risk based prices correct the maturity distortion for the group of borrowers who refinance. While FICO-based prices are more accurate than an entirely uniform interest rate, they still generate some (very small) maturity distortions and deadweight loss.

Counterfactual II: Transitioning to a Net Subsidy

The previous counterfactual showed that as risk-based pricing advances, low risk types will exit the public sector and the originally break-even rate will become an effective subsidy for the remaining borrowers. It is unclear whether the original value of $g$ minimizes the cost of this subsidy. In fact, our model highlights how policy makers must consider behavioral responses on two budget relevant margins when setting $g$: maturity choice and refinancing decisions. These responses change the costs associated with charging any given interest rate $g$.

The graphs in Figure 10 analyze how subsidy costs vary for the government with $g$, from FICO-based pricing and exited the public pool.
allowing for various behavioral responses by borrowers. The horizontal axis represents the different values of a uniform interest rate \( g \) that the government could charge on federal loans. On the vertical axis I plot the amount that the government will raise and spend on the federal loan portfolio for any value of \( g \), accounting for heterogeneous risk types, maturity choices, and refinancing choices. Therefore, the net interest rate tax or subsidy provided by the government at any given value of \( g \) can be found by tracing out the vertical distance between the dotted revenue line and the solid cost line at that point. The value of \( g \) that will allow the government to break even on the federal portfolio will be the point where the dotted line and the solid effective cost line intersect.

Panel (a) plots the average cost of the portfolio absent all maturity and refinancing responses (using the maturity choices of borrowers charged risk-based interest rates). This is equivalent to a policy scenario in which the government forced individuals into a specific maturity, and shut down the refinancing channel. Because there are no maturity or refinancing responses, this cost stays constant for all values of \( g \). If the government charged \( g = 6.15\% \), the point where cost is equal to revenue, they would break-even on the portfolio.

Panel (b) plots the average cost of the portfolio allowing only for a maturity response, not a refinancing response. This is equivalent to our baseline scenario, in which all borrowers were pooled in the federal portfolio. Note that as the government charges a lower \( g \), individuals extend their maturity which increases the average cost. As they increase \( g \) individuals decrease their maturity, which decreases the average cost. The break-even point is slightly higher, \( g = 6.39\% \), then in the case where there was no maturity response.

Panel (c) plots the average cost of the portfolio allowing only for a refinancing response, not a maturity response. The average cost curve is always upward sloping in \( g \), since as \( g \) increases the lowest risk individuals in the pool will refinance, increasing the average cost of the remaining pool. This means that it will be impossible to break-even.
on the portfolio once the refinancing channel is open, but it is possible to minimize the size of the subsidy provided to the remaining borrowers. At $g = 7\%$, the subsidy is minimized at 0.22%; moving from $g = 6.39\%$ to $g = 7\%$ reduces the interest rate subsidy by .12% from 0.34% to 0.22%.

Panel (d) plots the average cost of the portfolio allowing for both a refinancing response and a maturity response, the scenario closest to current policy. Individuals, especially higher risk individuals, choose a longer maturities under the flat, uniform $g$ price schedule. This increases the effective subsidy size for smaller values of $g$, and suggests that the government could minimize the cost of this subsidy by increasing the uniform rate slightly above the rate that assumes away all behavioral responses, to 8.27%.

7 Conclusion

Risk based pricing has advanced in many lending and insurance markets, with real implications for consumer surplus and the coexistence of private and public markets. In the private student loan origination market in 2011, 40% of new borrowers had FICO scores greater than 770, while less than 5% had scores below 670. At times, the government acts as a concurrent source of credit or a regulatory body: in the mortgage market, FHA-backed loan eligibility is predicated on risk-related factors like credit score, with government-provided subsidies for lower income households. In the health insurance market, the government has limited the set of risk-related factors that can determine premiums. This paper provides a framework to analyze and inform the role of government in such settings.

I study how the evolution of risk-based pricing in the student loan market impacts borrower welfare and government revenue. I show that without a private refinancing option, the governments’ uniform interest rate policy achieves modest redistribution over income, but generates sizable distortions in borrowers’ intertemporal consumption
Figure 10: Federal Loan Portfolio Cost vs. Revenue by Uniform Interest Rate, Accounting for Behavioral Responses

Notes: These graphs analyze how revenue and costs vary for the government with the uniform interest rate that they charge, $g$, and with various behavioral responses by borrowers. The horizontal axis represents the different values of a uniform interest rate $g$, that the government could charge and collect on federal loans. On the vertical axis, I show how the amount that the government will raise and spend on the federal loan portfolio, accounting for heterogeneous risk types, maturity choices, and refinancing choices, varies with $g$. The dotted lines, on the 45 degree angle, plots the uniform interest rate being charged. This is effectively the revenue the government will collect for a given value of $g$. The solid lines plot the average cost the government faces for a given value of $g$. Therefore, the net interest rate tax or subsidy provided by the government at any given value of $g$ can be found by tracing out the vertical distance between the dotted line and the solid line at that point. The value of $g$ that will allow the government to break even on the federal portfolio will be the point where the dotted line and the solid effective cost line intersect.

(a) Costs vs. Revenue by $g$, no Responses

(b) Costs vs. Revenue by $g$, with Maturity Response

(c) Costs vs. Revenue by $g$, with Refinancing Response

(d) Costs vs. Revenue by $g$, with Maturity and Refinancing Response
decisions. By refinancing into a risk-based interest rate, the average low-risk borrower can increase surplus by $1530, but the government will need to subsidize the remaining high-risk borrower on average $1507 to maintain equity.

In the student loan space, a subsidy policy will prevent unraveling and fulfill more precise redistributive motives – rather than implicitly “taxing” low risk borrowers under a break-even interest rate, the transfer could be funded by an income tax, by individuals who are not necessarily borrowers, and allow for intergenerational redistribution. In contrast, the findings suggests that a regulatory policy would primarily reduce efficiency, not inequity. I show that when firms can price only on FICO score, they reduce the gains to low risk borrowers substantially but do not reduce the extent of selection into the private market.

In addition to policy takeaways related to refinancing, this paper presents a novel micro-analysis of student loan repayment. It shows there is demand for flexible repayment structures (like a maturity continuum) that allow households to distribute payments optimally over time, and considerable heterogeneity in borrowers’ desire to lower monthly payments vs. interest rates. While this analysis focuses on how interest rates impact repayment decisions, student borrowers could also respond to interest rate levels at earlier steps in the borrowing process, for example when taking out debt or deciding whether to attend graduate school. The availability of better risk-based interest rates could change these decisions, and is an interesting area for future work.

While this analysis focuses on cross-sectional redistribution under a uniform rate, the policy also redistributes consumption longitudinally— all borrowers were liquidity constrained when beginning school and unable to secure private credit at price below the Federal rate. Thus even ex-post low risk borrowers benefit from the uniform rate in the interim. The welfare implications are more complex in light of this dynamic selection problem.
Bibliography


A Additional Figures and Tables

Figure 11: Distribution of calculated values of $\hat{P}(T)$

![Distribution of calculated values of $\hat{P}(T)$](image)

Figure 12: Variation in Observable Characteristics over Time

Notes: This graph shows changes in three important observable characteristics, income, debt amount, and FICO score, over 10 price regimes. While there are differences across price regimes, it is comforting to note that there are no obvious monotonic trends in these three variables and that they are not correlated with the exogenous price shifts.

![Variation in Observable Characteristics over Time](image)
Table 7: Maturity Choice Regression

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed 60</td>
<td>-4.506</td>
</tr>
<tr>
<td></td>
<td>(3.313)</td>
</tr>
<tr>
<td>Fixed 120</td>
<td>-15.32</td>
</tr>
<tr>
<td></td>
<td>(5.149)</td>
</tr>
<tr>
<td>FICO</td>
<td>-0.0706</td>
</tr>
<tr>
<td></td>
<td>(0.0156)</td>
</tr>
<tr>
<td>1 Risk Score</td>
<td>-21.58</td>
</tr>
<tr>
<td></td>
<td>(10.16)</td>
</tr>
<tr>
<td>2 Risk Score</td>
<td>-26.16</td>
</tr>
<tr>
<td></td>
<td>(10.53)</td>
</tr>
<tr>
<td>3 Risk Score</td>
<td>-35.07</td>
</tr>
<tr>
<td></td>
<td>(11.16)</td>
</tr>
<tr>
<td>4 Risk Score</td>
<td>-40.13</td>
</tr>
<tr>
<td></td>
<td>(11.78)</td>
</tr>
<tr>
<td>5 Risk Score</td>
<td>-45.29</td>
</tr>
<tr>
<td></td>
<td>(12.45)</td>
</tr>
<tr>
<td>6 Risk Score</td>
<td>-44.88</td>
</tr>
<tr>
<td></td>
<td>(13.20)</td>
</tr>
<tr>
<td>7 Risk Score</td>
<td>-45.35</td>
</tr>
<tr>
<td></td>
<td>(13.73)</td>
</tr>
<tr>
<td>8 Risk Score</td>
<td>-48.31</td>
</tr>
<tr>
<td></td>
<td>(14.13)</td>
</tr>
<tr>
<td>9 Risk Score</td>
<td>-52.13</td>
</tr>
<tr>
<td></td>
<td>(14.79)</td>
</tr>
<tr>
<td>10 Risk Score</td>
<td>-59.02</td>
</tr>
<tr>
<td></td>
<td>(15.51)</td>
</tr>
<tr>
<td>11 Risk Score</td>
<td>-55.87</td>
</tr>
<tr>
<td></td>
<td>(16.04)</td>
</tr>
<tr>
<td>12 Risk Score</td>
<td>-59.70</td>
</tr>
<tr>
<td></td>
<td>(16.59)</td>
</tr>
<tr>
<td>ln(amount)</td>
<td>25.46</td>
</tr>
<tr>
<td></td>
<td>(0.638)</td>
</tr>
<tr>
<td>Monthly FCF</td>
<td>-0.00263</td>
</tr>
<tr>
<td></td>
<td>(0.000284)</td>
</tr>
<tr>
<td>BA/BS</td>
<td>-1.143</td>
</tr>
<tr>
<td></td>
<td>(4.032)</td>
</tr>
<tr>
<td>Masters</td>
<td>-12.80</td>
</tr>
<tr>
<td></td>
<td>(4.072)</td>
</tr>
<tr>
<td>Professional</td>
<td>-13.28</td>
</tr>
<tr>
<td></td>
<td>(4.088)</td>
</tr>
<tr>
<td>Number Mortgages</td>
<td>2.390</td>
</tr>
<tr>
<td></td>
<td>(0.839)</td>
</tr>
<tr>
<td>Constant</td>
<td>19.99</td>
</tr>
<tr>
<td></td>
<td>(43.61)</td>
</tr>
</tbody>
</table>

N = 10680

Standard errors in parentheses
Regression also controls for dummy aged, and binned income. Omitted degree is BA. The firm-specific risk score was split into 13 bins, and these dummies are represented by the categories "1 Risk Score" etc, where 1 is the highest risk category and 12 is the lowest.

* p < 0.10, ** p < 0.05, *** p < 0.01
<table>
<thead>
<tr>
<th>Observed Variables</th>
<th>Definition</th>
<th>Data Used</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_0$</td>
<td>Annual Income</td>
<td>Observed Post-tax Annual Income</td>
<td>81,860</td>
<td>70,356</td>
<td>44,556</td>
</tr>
<tr>
<td>$d(T)$</td>
<td>Yearly Debt Payment at Maturity $T$</td>
<td>Yearly Student Loan Payment at Maturity $T$</td>
<td>11,005</td>
<td>7,980</td>
<td>9,006</td>
</tr>
<tr>
<td>$T$</td>
<td>Loan Maturity</td>
<td>Maturity in Months</td>
<td>85</td>
<td>60</td>
<td>44</td>
</tr>
<tr>
<td>$r(T)$</td>
<td>Risk-based Interest rate at Maturity $T$</td>
<td>Observed $r(T)$ %</td>
<td>4.95</td>
<td>5.04</td>
<td>0.90</td>
</tr>
<tr>
<td>$i(T)$</td>
<td>Risk-Free Interest Rate at Maturity $T$</td>
<td>Lowest Risk Type $r(T) - \epsilon$ in %</td>
<td>3.88</td>
<td>3.50</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Definition</th>
<th>Calibration Value Used</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Recovery Rate on Per-Period Payment</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Min. Consumption Threshold</td>
<td>Yearly Housing Expense</td>
<td>18,414</td>
<td>16,032</td>
<td>13,736</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>Definition</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(T)$</td>
<td>Per-Period Delinquency Probability</td>
<td>0.046</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>Standard Deviation of $\ln(w)$</td>
<td>0.57</td>
<td>0.29</td>
<td>1.12</td>
</tr>
</tbody>
</table>

*Table 8: Empirical Income Volatility Exercise: Calibration Values and Estimates*
Figure 13: Average Size of Extra Payments Over Time Made by Borrowers

Notes: I look at payment patterns over time within my sample of refinancers – in other words, do any individuals change their payment level over time permanently, or do they systematically make higher or lower payments on their debt. I find that there are some extra payments in the data, but they are small and do not vary systematically over time. This supports our model’s assumption that borrowers make a term choice in year 1 to maximize expected utility over the life of the loan and are not in fact choosing a monthly payment to fit their current income level, with the intent to refinance and change term yet again in the future when their income level changes.

![Graph showing deviations from set payment schedule over time](image)

Figure 14: Levels and Changes in Other Monthly Payments Before and After Refinancing

Notes: My model assumes that borrowers are not readjusting on other financial margins when refinancing. In other words, contemporaneous savings and debt decisions are assumed to be exogenous, predetermined, and unaffected by maturity and refinancing decisions. Here I test this assumption by looking at borrowers’ other monthly payments before and after refinancing. This table describes changes in other monthly payments (mortgages, auto loans, credit cards, etc) before vs. after refinancing for individuals who had positive monthly payments to begin with, and shows that for the vast majority of borrowers these stayed constant. This makes sense, since many of these payments are fixed installments, and it would take active work on the borrower’s part to readjust.

<table>
<thead>
<tr>
<th>Initial Monthly Payment</th>
<th>Initial Auto</th>
<th>Initial Real Estate</th>
<th>Initial Non-Real Estate</th>
<th>Initial Credit Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>449.32</td>
<td>1927.72</td>
<td>1104.31</td>
<td>94.37</td>
</tr>
<tr>
<td>Median</td>
<td>386.00</td>
<td>1698.00</td>
<td>882.00</td>
<td>52.00</td>
</tr>
<tr>
<td>IQR (25,75)</td>
<td>246.50</td>
<td>1210.00</td>
<td>897.00</td>
<td>84.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Change in Monthly Payment</th>
<th>Auto Change</th>
<th>Real Estate Change</th>
<th>Non-Real Estate Change</th>
<th>Credit Card Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>37.15</td>
<td>63.64</td>
<td>-181.06</td>
<td>-12.74</td>
</tr>
<tr>
<td>Median</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>IQR (25,75)</td>
<td>0.00</td>
<td>37.00</td>
<td>490.00</td>
<td>52.00</td>
</tr>
</tbody>
</table>
Figure 15: Investment Balances over Lifetime

Notes: My model defines yearly consumption as income minus the student debt payment; in reality individuals may also be making savings decisions that could impact their maturity choices. I can observe the savings and investment behavior of borrowers in my sample: because individuals in my sample are young, they have relatively low levels of savings to begin with. Slightly under 40% have a formal retirement savings account – for example 25% have a 401k, with a median balance of $24,000. The number of individuals with investment holdings increases with age. This figure shows that while the median borrower continues to not have substantial savings through age 60, the 75th percentile has accumulated over $80,000 by age 50. However, 90% of my borrowers are under 40 years old, and therefore even the most active savers have investment holdings that are much smaller than their student debt amount.
B Derivation of Analytical First Order Condition:

Analytical Estimation:
We observe starting income levels \( w_{i0} \). This means that we can express income at time \( t \) as:

\[
\ln(w_{it}) = \ln(w_{i0}) + t \cdot (X_i'\mu) + \sum_{1}^{t} u_{it}
\]

\[w_{it} = w_{i0} \cdot e^{t \cdot (X_i'\mu) + \sum_{1}^{t} u_{it}}\]

If we return to the uncertain portions of the right hand side of our first order condition, \( E[(w_{it} - d_{i})^{-\gamma}] \), note that we can rewrite the expected marginal utility as the marginal utility of a certainty equivalent given by:

\[
E[(w_{i0} \cdot e^{t \cdot (X_i'\mu) \cdot e^{\sum_{1}^{t} u_{it}}} - d_{i})^{-\gamma}] = (w_{i0} \cdot e^{t \cdot (X_i'\mu) \cdot e^{\sum_{1}^{t} u_{it}}} - d_{i})^{-\gamma}
\]

where \( \pi_{it} \) is the certain amount an individual would have to be given in that period to make their certain utility equivalent to the expected utility. Specifically:

\[
\pi_{it} = \begin{cases} 
\frac{1}{2} \cdot t \cdot \sigma^2 \left[ 1 - (1 + \gamma) \right] \frac{w_{i0} \cdot e^{t \cdot (X_i'\mu)}}{w_{i0} \cdot e^{t \cdot (X_i'\mu)} - d_{i}} & \text{for } t < T + 1 \\
\frac{1}{2} \cdot t \cdot \sigma^2 (-\gamma) & \text{for } t \geq T + 1
\end{cases}
\]

To derive \( \pi_{it} \), note that we can write:

\[
E[u'(w_{it})] = E[u'(w_{i0} \cdot e^{t \cdot (X_i'\mu) \cdot e^{\sum_{1}^{t} u_{it}}})]
\]

56
where $\epsilon_{it} \sim N(0, 1)$. We want to find the value of $\pi(\sigma)$ that allows us to write:

$$E[u'(w_{i0} * e^{\epsilon_{i}X_{i}u}) * e^{\sum_{t \geq 1} \epsilon_{it} - d_{i}}] = u'(w_{i0} * e^{\epsilon_{i}X_{i}u} * e^{\pi(\sigma)} - d_{i})$$

For simplicity, start with the case of no income growth in period 1.

$$E[u'(w_{i0} * e^{\sigma_{i1} - d_{i}})] = u'(w_{i0} * e^{\pi(\sigma)} - d_{i})$$

We first take the derivative of this expression w.r.t. $\sigma$:

$$E[w_{i0} * e^{e_{i1} u''(w_{i0} * e^{\sigma_{i1} - d_{i}})] = \pi'(\sigma)w_{i0} * e^{\pi(\sigma)} u''(w_{i0} * e^{\pi(\sigma)} - d_{i})$$

At $\sigma = 0$ this becomes zero since $E[\sigma e] = 0$ and thus $\pi'(0) = 0$.

We next take the second derivative of this expression w.r.t. $\sigma$, and evaluate it at $\sigma = 0$:

$$E[e^{2}u''(w_{i0} - d_{i}) + e^{2}w_{i0}u'''(w_{i0} - d_{i})] = \pi''(0) u''(w_{i0} - d_{i})$$

$$\pi''(0) = \left[1 + \frac{u''(w_{i0} - d_{i})}{w_{i0} - d_{i}}\right]$$

Under the assumption of CRRA utility, this becomes:

$$\pi''(0) = \left[1 + \frac{u''(w_{i0} - d_{i})}{w_{i0} - d_{i}}\right]$$

$$= \left[1 - (1 + \gamma) \frac{w_{i0}}{w_{i0} - d_{i}}\right]$$

We now have a value for $\pi''(0)$. This is helpful when evaluating a Taylor expansion of $\pi(\sigma)$:

$$\pi(\sigma) \approx \pi(0) + \pi'(0)\sigma + \frac{1}{2} \sigma^{2} \pi''(0)$$

$$\pi(\sigma) \approx \frac{1}{2} \sigma^{2} \left[1 - (1 + \gamma) \frac{w_{i0}}{w_{i0} - d_{i}}\right]$$

57
C  Online Appendix: Modeling Borrower Delinquency

Borrower problem: Impact of delinquency risk

In this section I solve the borrowers’ problem, allowing them to consider the impact of delinquency on their expected utility. Borrowers of risk type $p$ and debt amount $D$ choose maturity $T$ to maximize:

$$
\max_T \sum_{t=1}^{Q} \mathbb{E}[u(c(t))]
$$

where $Q$ is the individuals’ maximum age (i.e. finite). I define consumption as post-tax income $w_t$ minus a student debt payment $d(T)$ if the loan is still being paid off.

The debt payment $d(T)$, that individuals make each period is a function of their total debt amount $D$, their chosen maturity, $T$, and the risk, maturity-specific interest rate schedule that they are offered, $r(T,p)$:

$$
d(T) = D \frac{r(T,p)}{(1 - (1 + r(T,p))^{-T})}
$$

In the borrowers’ maximization problem, agents can decrease their debt payment by increasing their maturity – i.e. $\frac{dd(T)}{dT} < 0$. This is the only “choice” variable that the borrower has – I assume that the interest rate schedule a borrower is offered is exogenous (determined by the lender) and therefore cannot be manipulated by the borrower to change their monthly payments or financing costs.

While borrowers can control their yearly payment level, they cannot control their variable, growing income stream which I parametrize as a unit root process (as in the main specification). Since income is variable, it is possible that a bad income shock could force consumption as currently defined $(w_t - d(T))$ to a negative level. Therefore
I specify a certain minimum threshold \( x \) that consumption never falls below. If ever \( (w_t - d(T) < x) \), an individual will not pay their entire debt payment in that period and instead consume \( x \), creating a discontinuity in the consumption function. Specifically, the consumption function is given by:

\[
c(t) = \begin{cases} 
(w_t - d(T)) \cdot 1(w_t - d(T) \geq x) + x \cdot 1(w_t - d(T) < x) & \text{for } t \leq T \\
w_t & \text{for } t > T
\end{cases}
\]

If a borrower ever misses a payment (or part of a payment) in a period, the borrower will have to continue making payments on the loan as scheduled for the remainder of the \( T \) periods and face a large utility penalty, which I denote as \( G \), after \( T \). If \( G \) is large enough, this will rule out the possibility of “strategic” default on the borrowers’ part.\(^{30}\) If an individual never misses a payment, they will consume their entire income in every period after period \( T \) and not face a penalty.

I assume that both the firm and the borrower have symmetric expectations about the borrowers’ income process, and therefore of \( Pr(w_t - d(T) < x) \). In the following section, I empirically back out the \( Pr(w_t - d(T) < x) \) that is implied from each borrower/maturity specific interest rate that is offered by the firm.

To simplify the maximization problem, I make a key assumption that the probability of delinquency does not change over time and is equal to the borrower’s initial delinquency.\(^{30}\) Specifically, \( G \) must be large enough that totally utility after missing a payment is smaller than total expected utility without missing a payment.

\(^{30}\)Specifically, \( G \) must be large enough that totally utility after missing a payment is smaller than total expected utility without missing a payment.
quency risk; i.e. \( Pr(w_t - d(T) < x) = Pr(w(0) - d(T) < x) = P(T) \). While \( P(T) \) does not change over time, I still allow the probability of delinquency to change with the chosen maturity; specifically, \( \frac{dP(T)}{dT} < 0 \), since a longer maturity will lower the the monthly payment in any given period.

I also make the delinquency penalty \( G \) linearly additive to utility from consumption. These simplifications make the maximization problem:

\[
\begin{align*}
&= \max_T \sum_{t=1}^{T} \left( \text{EU in period if not delinq.} \cdot u(w_t - d(T)) | w_t - d(T) > x \right) \cdot (1 - P(T)) + u(x) \cdot P(T) + \\
&\quad \sum_{t=T+1}^{Q} \left( \text{EU if never delinq.} \cdot u(w_T) \right) - G \cdot P(T) \cdot T
\end{align*}
\]

Differentiating the borrower problem with respect to \( T \), we can see the exact impact of a change in maturity on expected utility through the first order condition. Allowing delinquency to enter the problem also adds two additional terms to the simplified first order condition.

\[Pr(w(0) - d(T) < x) = Pr(w(0) < x + d(T))\]

\[= Pr(ln(w(0)) + u_t < ln(x + d(T)))\]

\[= Pr(u(t) < ln(x + d(T)) - ln(w(0)))\]

\[= \frac{\Phi(ln(x + d(T)) - ln(w(0)))}{\sigma_u(p)}\]
order condition:

\[ 0 = \sum_{t=1}^{T} \mathbb{E}[-u'(w_t - d(T))|w_t - d(T) > x] \times \frac{dd(T)}{dT} \times (1 - P) \]

\( A: \) Increase in utility from having a slightly lower payment each period

\[ + \sum_{t=1}^{T} (u(x) - \mathbb{E}[u(w_t - d(T))|w_t - d(T) > x]) \times \frac{dP}{dT} \]

\( B: \) Change in EU due to change in probability of being delinquent

\[ + \left( \mathbb{E}[u(w_T - d(T))|w_T - d(T) > x] \times (1 - P) + u(x) \times P \right) - \mathbb{E}[u(w_T)] \]

\( C: \) Decrease in utility from having to pay for one additional period

\[ - G \times (P(T) + T \times \frac{dP(T)}{dT}) \]

\( D: \) Change in expected penalty payment due to longer loan period

An increase in maturity will now impact expected utility in four main ways:

- It lowers the size of the monthly payment, which increases utility while paying off the loan (Effect A). Because agents are hand-to-mouth, they can better smooth consumption with a lower monthly payment – there will be a small “jump” in utility when they finish paying off the loan. A second utility smoothing benefit comes from the fact that income is risky and a lower monthly payment provides some “insurance” value against more volatile income shocks. However, the interest rate \( r(T) \) increases with maturity, so \( d(T) \) is decreasing in \( T \) at a decreasing rate.

- It changes the length of repayment, which means the borrower has to pay the loan off for one additional period, and this lowers total expected utility (Effect C).

- It decreases the probability of delinquency, \( P(T) \), in any single period since there is a lower monthly payment. \( During \) repayment, this increases expected utility by lowering the chance that you consume at the threshold \( x \) (Effect B). \( After \) repayment, this increases expected utility by lowering the chance that you have to
pay the penalty $G$ (Effect D).

- It increases the probability of delinquency over the life of the loan since you are paying off over more periods. This increases the expected penalty $G$ (Effect D).

The main specification captures the consumption smoothing effects, $A$ and $C$, of a lower maturity; the delinquency threshold adds the additional effects, $B$ and $D$. The FOC allows one to say something about the size and sign of these effects: the size of effect $B$ is bounded mechanically – when $(w_t - d(T))$ is small, $(\mathbb{E}[u(w_t - d(T)) | w_t - d(T) > x] - u(x))$ is close to zero. Conversely, when $(w_t - d(T))$ is large, there is a small probability of delinquency and $\frac{dP}{dT}$ is near zero. This means effect $B$ is positive but small. Effect $D$ has an ambiguous sign and size. As maturity increases, each period the chance of delinquency becomes smaller; yet, the number of periods during which delinquency can occur increases. This means that $(P(T) + T * \frac{dP(T)}{dT})$ could be positive or negative. The size of $D$ also depends on the size of $G$, the delinquency “penalty”, which is unobserved.

In the empirical section I test how sensitive estimates of $\gamma$ are to the inclusion or exclusion of effect $D$. I calibrate $(P(T) + T * \frac{dP(T)}{dT})$ using the empirical delinquency probabilities implied by observed risk-based interest rates, and test the sensitivity of $\gamma$ to a wide range of values of $G$. The results suggests that the estimates are not very sensitive to the exclusion of the delinquency channel – for this low risk group, the first order effect of a change in maturity seems to operate instead through the monthly payment consumption smoothing channel.